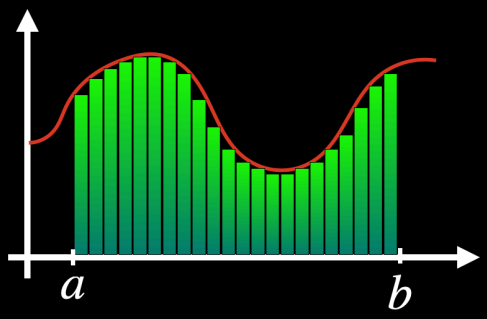
There are 33 different types of integrals in mathematics (as far as we could find). Of course, mathematics is constantly evolving, so there are probably many more out there. Beyond that, many of the integrals in this list are particular cases, or generalizations, of one another. Anyway, this list gives a very good understanding of the overall picture of integration in mathematics.

So, even if this list that we are about to present to you is not complete, it is a *pretty* extensive one! Let us know in the comment section which one you found most interesting. Here we go:

1. Riemann Integral

The most common integral, representing the area under a curve over a closed interval [a , b].

$$\int_a^b f(x)\,dx$$



2. Riemann-Stieltjes Integral

WHICH IS THE ONE WE WILL EXPLAIN IN DETAIL IN THE NEXT VIDEO...

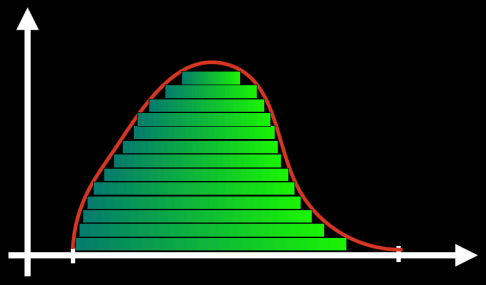
STAY TUNED...

(BTW, consider becoming a member of the channel!) Thanks!

3. Lebesgue Integral

Which is an integral that generalizes the Riemann integral by measuring the "size" of sets in the range of the function and integrating with respect to a measure.

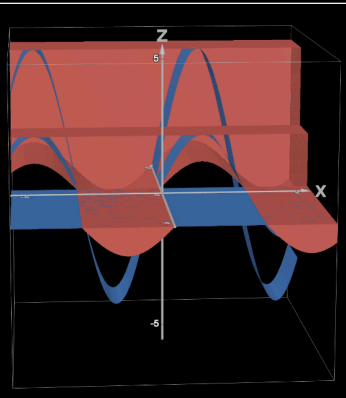
$$\int_E f(x)\,\mu(dx)$$



4. Lebesgue-Stieltjes Integral

It combines the concepts of Lebesgue integration and Stieltjes integration, allowing for integration with respect to functions of bounded variation.

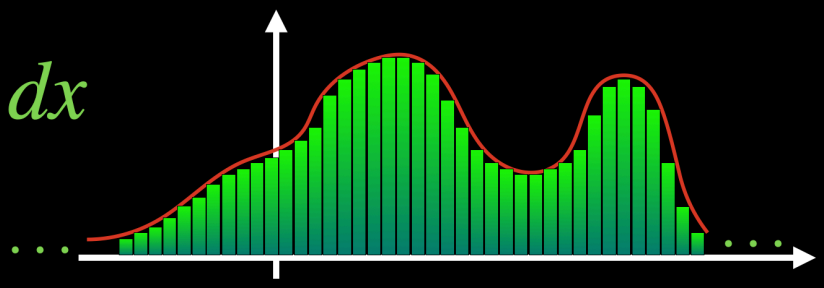
$$\int_a^b f(x)\,d\alpha(x)$$



5. Improper Integral

Extends the concept of the Riemann integral to unbounded intervals or functions with singularities.

$$\int f(x)\,dx$$



6. Multiple Integral

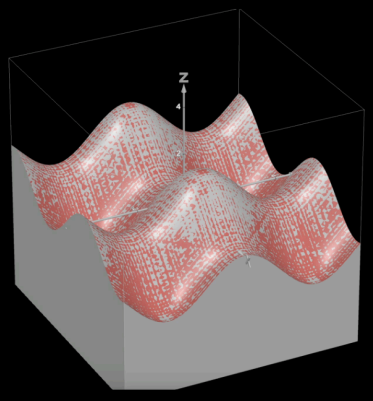
Integrals over functions of several variables, including:

- **Double Integral:** *for functions of two variables, representing volumes under surfaces.*

- **Triple Integral:** *for functions of three variables, representing volumes in higher-dimensional spaces.*

- **And so on for higher numbers of integrals.**

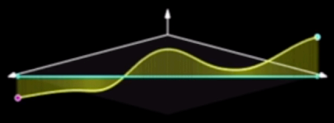
$$\int_{a_y}^{b_y}\int_{a_x}^{b_x} f(x,y)\,dx\,dy$$



7. Line Integral

Integrates a function along a curve or path in space, commonly used in physics for work, circulation, and flux.

$$\int_C f(\mathbf{r})\, ds = \int_a^b f(\mathbf{r}(t)) \left| \mathbf{r}'(t) \right| \, dt$$

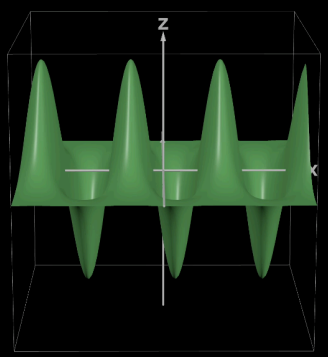


8. Surface Integral

It generalizes the line integral to integrate over a surface in three-dimensional space.

$$\iint_S f(\mathbf{r})\, dS =$$

$$\iint_D f(\mathbf{r}(u,v)) \left| \mathbf{r}_u \times \mathbf{r}_v \right| \, dudv$$



9. Volume Integral

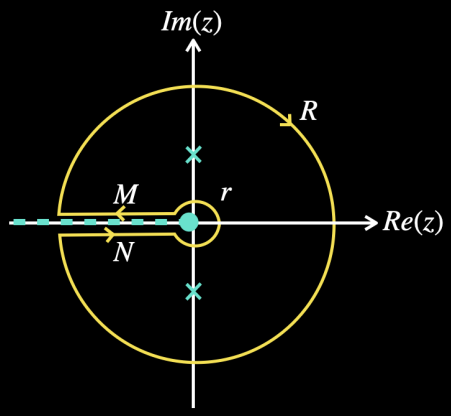
A generalization of the surface integral to integrate over a three-dimensional volume.

$$\iiint_V f(\mathbf{r})\, dV$$

10. Contour Integral

A complex integral where the function is integrated along a path in the complex plane.

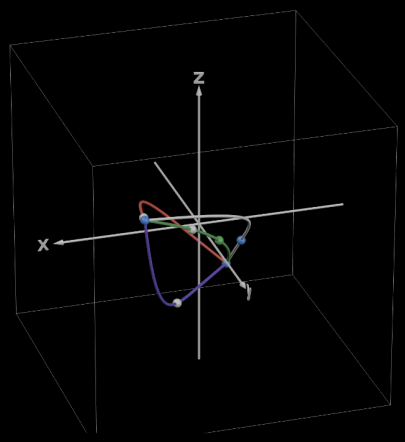
$$\oint_C f(z)\, dz$$



11. Path Integral

Used in physics, particularly quantum mechanics, integrating over all possible paths between two points.

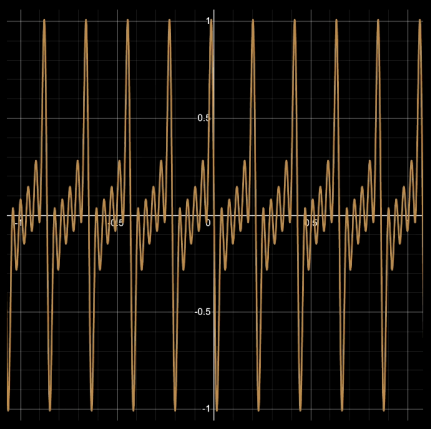
$$\int_C \mathbf{F} \cdot d\mathbf{r}$$



12. Fourier Integral

Decomposes a function into its sine and cosine components (related to the Fourier transform).

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i \xi x} \, d\xi$$



13. Laplace Integral

Related to the Laplace transform, used to solve differential equations by converting them into algebraic equations.

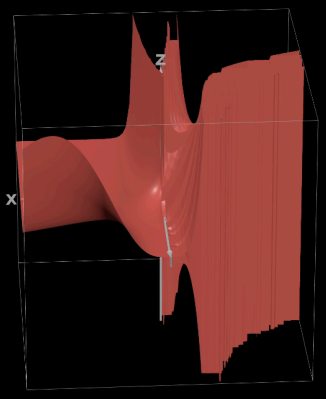
$$\mathcal{L}\{f(t)\} = F(s) =$$

$$= \int_0^\infty f(t)e^{-st}\,dt$$

14. Mellin Integral

Associated with the Mellin transform, often used in number theory and asymptotic analysis.

$$\mathcal{M}\{f(x)\}(s) = \int_0^\infty f(x)x^{s-1}\,dx$$

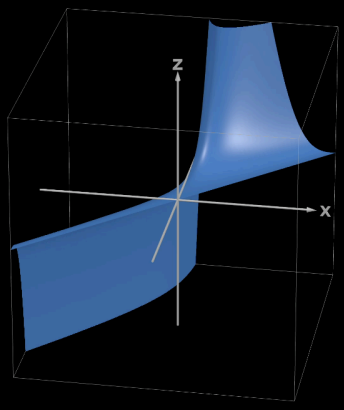


15. Fractional Integral

Generalizes the concept of an integral to non-integer orders, used in fractional calculus.

$$I^{\alpha} f(x) =$$

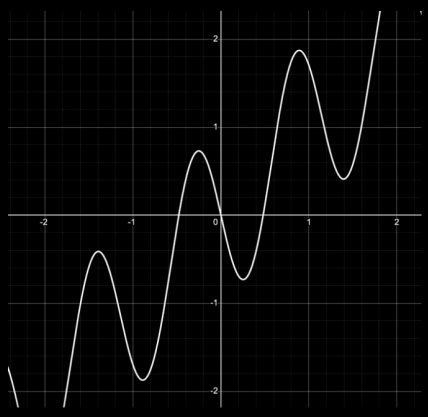
$$= \frac{1}{\Gamma(\alpha)} \int_0^x (x - t)^{\alpha - 1} f(t)\, dt$$



16. Stochastic Integral (Itô Integral)

Used in stochastic calculus, particularly in the study of stochastic processes and Brownian motion.

$$\int_0^T f(t)\, dW(t)$$



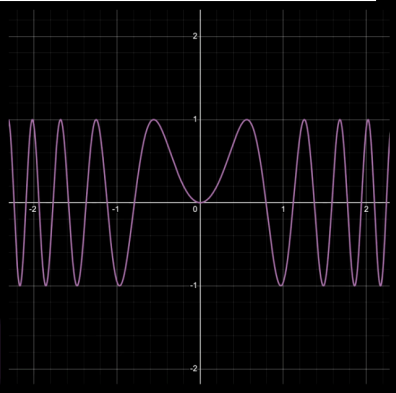
17. Henstock-Kurzweil Integral (or Gauge Integral)

It generalizes the Riemann integral in a way that can handle more functions without requiring Lebesgue integration.

$$\int_a^b f(x)\,dx =$$

$$= \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

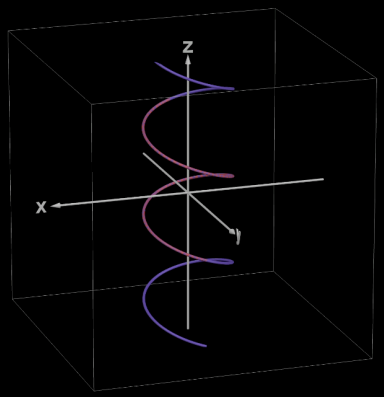
where $\Delta x_i \le \delta(x_i)$ — *gauge function*



18. Bochner Integral

It generalizes the Lebesgue integral to functions that take values in Banach spaces.

$$\int_a^b \vec{f}(t)\,dt$$



19. Daniell Integral

A generalization of the Lebesgue integral, formulated without reference to a measure, based on set functions.

$$I(f) = \int f(x)\, d\phi(x)$$

linear functional

20. Cauchy Principal Value

not quite an integral… but treats improper integrals as the symmetric limit around singularities, assigning a finite value to otherwise divergent integrals.

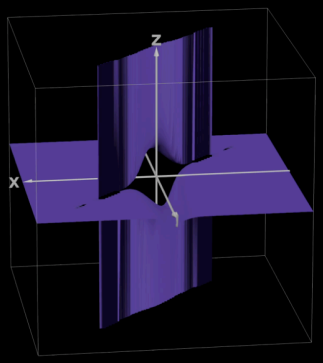
A method to assign values to certain divergent integrals, often used in complex analysis.

$$\text{P.V.} \int_a^b f(x)\, dx$$

21. Stieltjes Transformation

A type of integral transform similar to the Laplace and Fourier transforms, used in probability theory and mathematical physics.

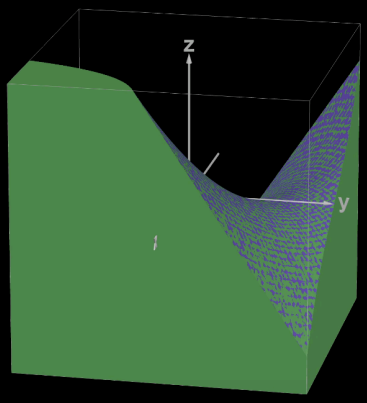
$$S_f(z) = \int_{-\infty}^{\infty} \frac{f(x)}{x - z}\, dx$$



22. Fredholm Integral

An integral equation where the solution appears under the integral sign and is integrated over a fixed range.

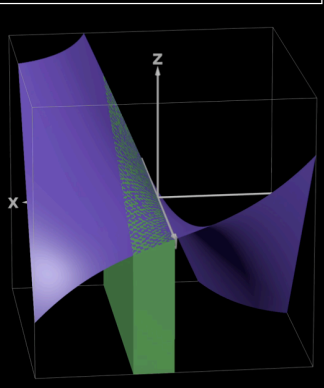
$$\varphi(x) = f(x) + \lambda \int_a^b K(x, t)\varphi(t)\, dt$$



23. Volterra Integral

Similar to the Fredholm integral but integrated over a variable range, used in various applied mathematics problems.

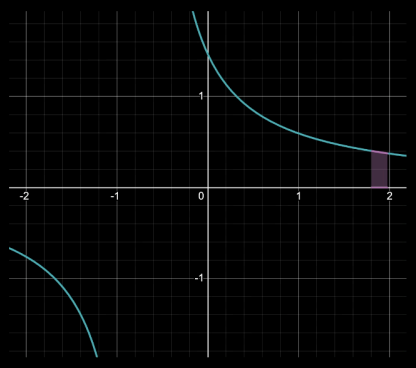
$$\varphi(x) = f(x) + \lambda \int_a^x K(x, t)\varphi(t)\, dt$$



24. Singular Integral

An integral in which the integrand has singularities, requiring specialized techniques for evaluation.

$$\text{P.V.} \int_a^b \frac{f(x)}{x - c}\, dx$$



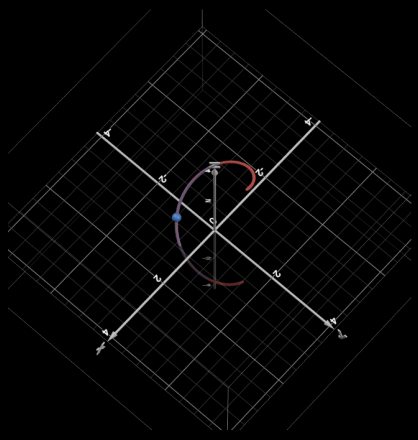
25. Pettis Integral

A generalization of the Lebesgue integral to vector-valued functions that need not be strongly measurable.

$$\int f(x)\, d\mu(x)$$

Banach space-valued function

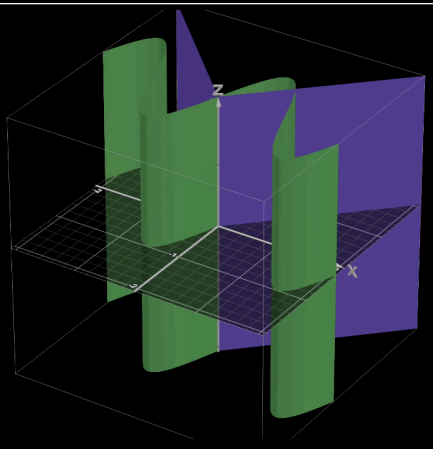
measure



26. Young Integral

A generalization of the Riemann-Stieltjes integral for functions of bounded p-variation.

$$\int_a^b f(x)\, dg(x)$$

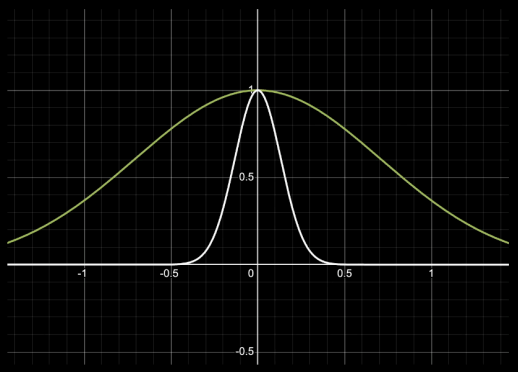


27. Haar Integral

Integral on topological groups, particularly important in harmonic analysis and representation theory.

$$\int_G f(g)\, d\mu(g)$$

Haar *measure*

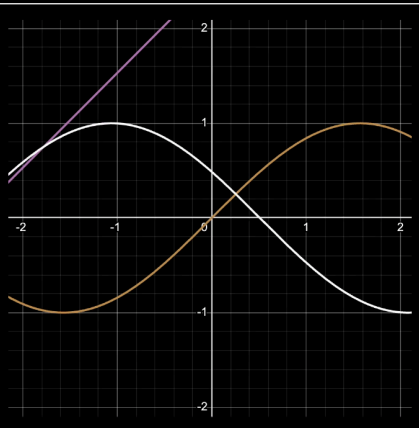


28. Birkhoff Integral

A variant of the Riemann-Stieltjes integral, used in ergodic theory and measure theory.

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x) =$$

$$= \int_X f(x)\, d\mu(x)$$

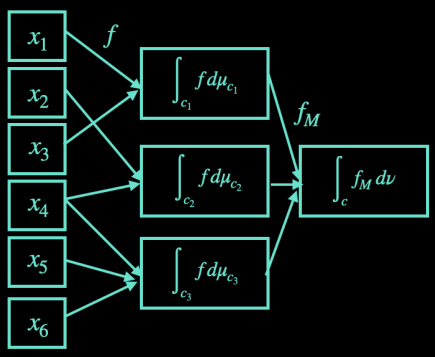


29. Choquet Integral

A type of integral that generalizes the Lebesgue integral, especially in the context of non-additive measures or fuzzy measures.

$$\int f \, d\nu =$$

$$= \int_0^\infty \nu(\{x : f(x) \geq t\}) \, dt$$

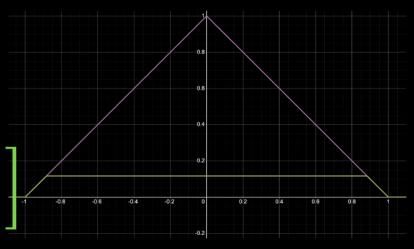


30. Sugeno Integral

Similar to the Choquet integral but used in decision theory, particularly in fuzzy logic and multi-criteria decision making.

$$\int_S^{\text{Sugeno}} f(x) \, d\nu =$$

$$= \sup_{t \in [0,\infty)} \left[\min \left(t, \nu(\{x : f(x) \geq t\}) \right) \right]$$

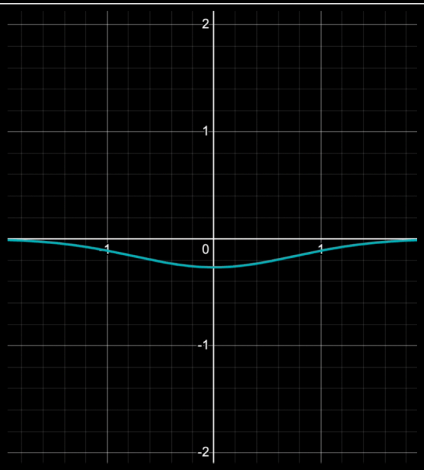


31. Schwarz Integral

Related to the theory of distributions, used in the analysis of generalized functions.

$$\int_{-\infty}^{\infty} \varphi(x)\,dx$$

Schwarz function



32. Wiener Integral

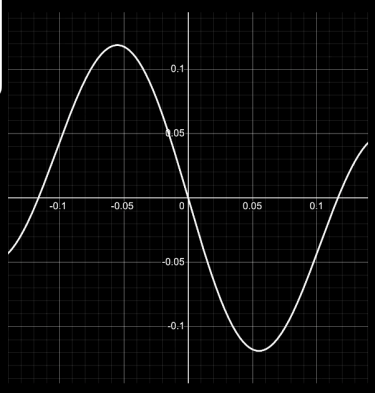
Used in the theory of stochastic processes, particularly in the analysis of Brownian motion.

$$\int_0^T f(t)\,dW_t$$

upper limit of the time interval

deterministic function

Wiener process (Brownian motion)



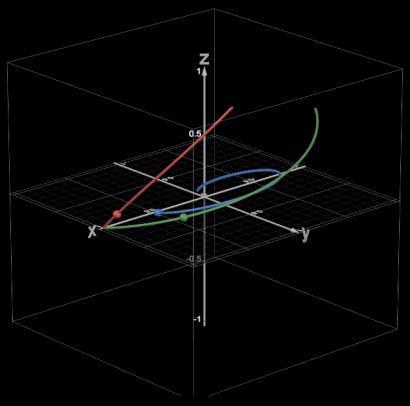
33. Feynman Integral

application

A type of path integral used in quantum mechanics and quantum field theory.

$$\langle x_f, t_f | x_i, t_i \rangle =$$

$$= \int \mathscr{D}[x(t)]\, e^{\frac{i}{\hbar} S[x(t)]}$$



As we said before, we don't have time to talk about all of these integrals here, so we will only study the Riemann-Stieltjes integral in the next video in the channel. But if you guys would like us to make a video explaining any of these other integrals, including the intuitions and visual representations of each, let us know in the comment section.

Comment: Please, if you find this document useful, let us know. Or if you found typos and things to improve, let us know as well. Your feedback is very important to us, since we are working hard every day to deliver the best material possible. Contact us at: dibeos.contact@gmail.com