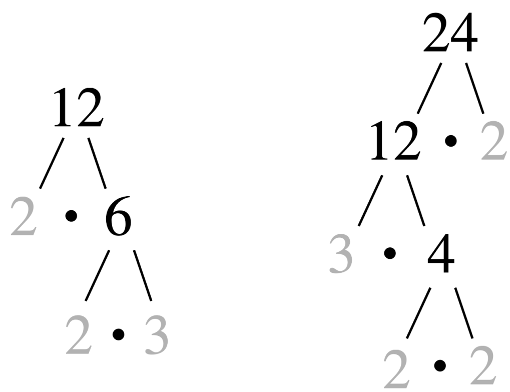
Primes

A number like 12 is known as a **composite number**. Other examples are 4, 6, 8, 9, etc. The other numbers we did not mention are the numbers which can only be written as a product of themselves, and are therefore prime numbers.

A precise definition is “a positive integer greater than 1 that cannot be evenly divided by any other integer except itself and 1”

Except for 0 and 1, every natural number is either composite or prime. Every single composite number can be foundationally factored into primes. And it can be done so in one way and one way only. So except for the order, we arrive at the same factors.



This is known as the **Fundamental Theorem of Arithmetics**. But – there are two reasons why this analogy is incomplete. First because there is an infinite number of primes.

How do we know that? Let’s assume the opposite – that there is a finite number of primes $p_1, p_2, p_3, \ldots$ up to p_n.

Say we want to multiply all of these primes together and create the number *P*. What would happen?

$$P = p_1 \cdot p_2 \cdot p_3 \cdot \ldots p_n$$

We know that *P* will absolutely have to be perfectly divisible by each and any p_i, since it is a factor of *P*. *P* doesn’t tell us anything new – it’s just a product of the previous numbers.

Things change when we add 1 to the set of prime numbers. The resulting number will be a new number, called N.

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$$N = p_1 \cdot p_2 \cdot p_3 \ldots p_n + 1$$

We know for a fact that if *N* results in a prime number, it will be a new prime and must therefore be added to the list of primes.

If *N* happens to be a composite number instead, we need to see if it breaks into new primes. Which by definition, it will. These resulting new primes should be added to the list.

For example, say we start with a list of $2 \cdot 3 \cdot 5 + 1$. *N* is therefore 31. 31 is a new prime number which is not already on the list, and must therefore be added to it.

Now we have a new set: $2 \cdot 3 \cdot 5 \cdot 31 + 1$. This generates the number 931, which is not a prime number.

But, it can be broken down into the primes $7 \cdot 7 \cdot 19$, both of which are not on the list and must be added to it.

By doing this process over and over again, we set off a “chain reaction” which generates infinite numbers, making our initial guess of the idea that there are finite prime numbers absurd.

The second thing is – there is no clear pattern to them. All the attempts to produce a prime producing formula have failed. But there are some helpful patterns.

Primes become more and more sparse the higher up we go. Although even that is an “on average” sort of thing because there are 17 primes between 400 and 500, and 16 primes between 200 and 300.

There has been an advancement in the statistical distribution of the primes, called the **Prime Number Theorem**.

It attempts to answer the question: given a positive integer n, how many integers up to, and including n, are prime numbers? The theorem looks like this:

$$\lim_{n \to \infty} \frac{\pi(n)}{\frac{n}{\ln(n)}} = 1$$

But, we will use this version for simplicity

$$\frac{n}{ln(n)}$$

For example, let's take the number 1000. If we plug it in we have:

$$\frac{1000}{ln(1000)} \quad \frac{1000}{6.9} \approx 145$$

The actual number of primes up to 1000 is actually 168. It's close. But the theorem tells us something about the approximation as a *percentage* of the true value.

The true value was 168 and the approximation 145. Therefore, the approximation has a proportion of 0.86.

$$\frac{145}{168} = 0.86$$

The higher *n* is the better the approximation will be.

The largest prime number found up to today was on October 12, 2024, it is $2^{136,279,841} - 1$ and has 41,024,320 digits.

The last pattern observed is that primes tend to sometimes crowd themselves in this form $p + 2$ where *p* is a prime number.

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