Every Fundamental Branch of Pure Math in 16 Minutes
By DiBeos

Mathematics is built on foundational areas like geometry, calculus, algebra, real analysis, etc. But these fields can all be broken down into specialized branches of mathematics, which can be in one specific field or be shared by several. Let's go through every one of them.

1. **Algebraic Number Theory**

This branch explores the properties and relationships of complex numbers that satisfy polynomial equations with integer coefficients. Algebraic numbers include all rational numbers, roots of unity, and solutions to polynomial equations like $x^n + a_{n-1}x^{n-1} + \dots + a_0 = 0$, where the coefficients a_i are integers. Algebraic number theory studies many kinds of structures and systems that can be derived from these numbers, like number fields and ring properties. The theory also goes into ideals, units, and the unique factorization of ideals in rings of algebraic integers.

number fields

$$\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$$

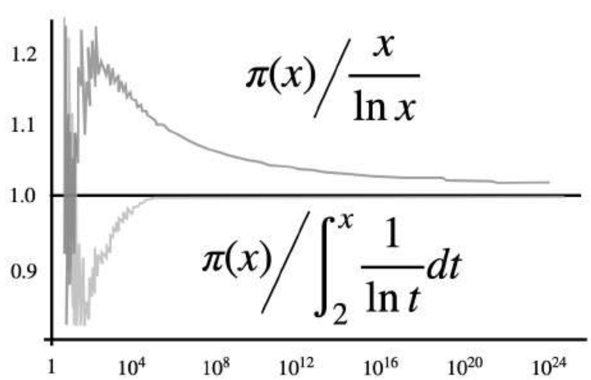
ring properties

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

See book on Algebraic Number Theory

2. **Analytic Number Theory**

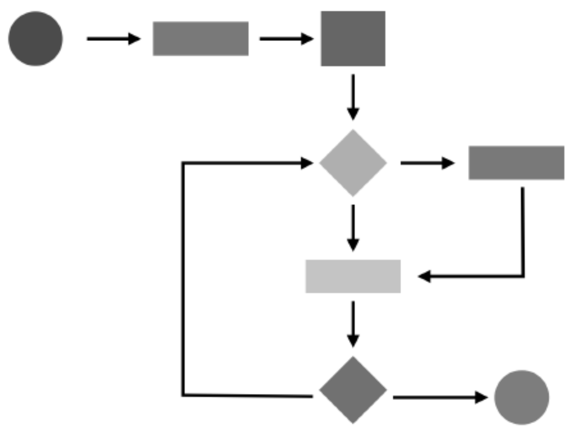
Analytic Number Theory mostly focuses on approximations, instead of the exact solutions. It involves the deep analysis of number distributions, specifically speaking – primes. This branch of number theory does not expect exact formulas for most of its quantities, unless they are constructed artificially.



See book on [Analytic Number Theory]

3. Computational Number Theory

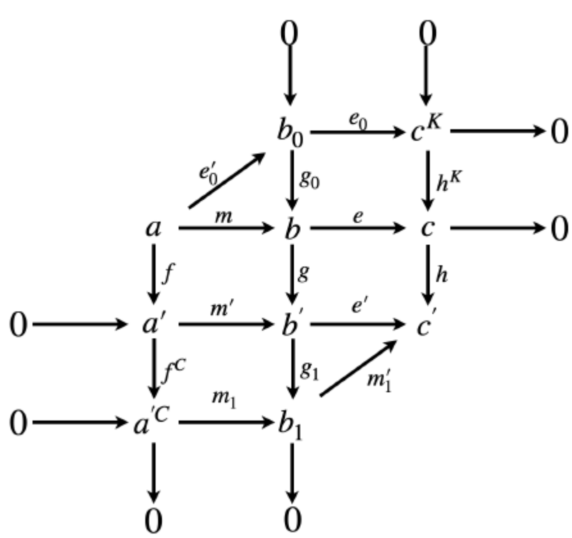
This branch of number theory focuses on developing algorithms to solve numerical problems related to integers and other number-theoretic issues. It involves the creation and application of computational methods to perform calculations on large numbers, like factoring integers, finding primes, and computing discrete logarithms.



See book on [Computational Number Theory]

4. Abstract Algebra

Abstract algebra examines the properties and structures of algebraic systems like groups, rings, and fields. And does so through concepts like homomorphisms, isomorphisms, and ideals. This field has many sub branches.

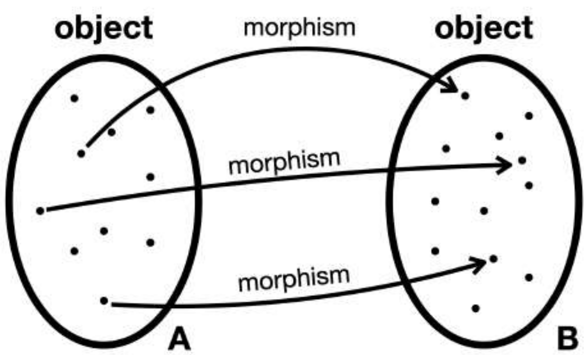


See book on [Abstract Algebra]

Watch video about [Abstract Algebra] Abstract Algebra is Impossible Without These 8 Things

5. **Category Theory**

The main idea in category theory is to focus on the relationships between objects rather than the objects themselves. At its core, a category consists of objects and morphisms (also called arrows or maps) between those objects. Morphisms can be thought of as processes or transformations from one object to another, satisfying properties like associativity and the existence of identity morphisms.

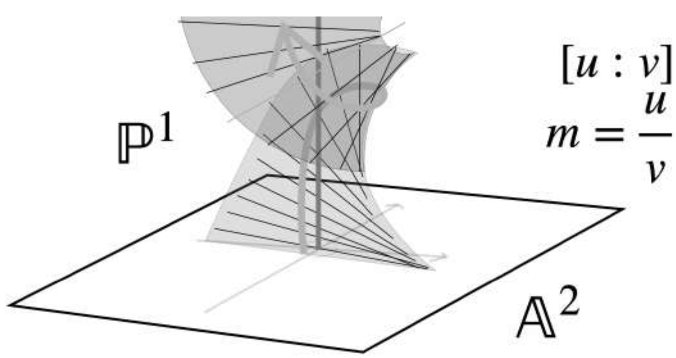


See book on [Category Theory]

Watch video about [Category Theory] Category Theory is Impossible Without These 6 Things

6. **Algebraic Geometry**

The branch of mathematics that studies properties and relationships of geometric structures that can be described algebraically through polynomial equations. This field combines abstract algebraic concepts, such as rings and fields, with geometric intuition.



See book on [Algebraic Geometry]

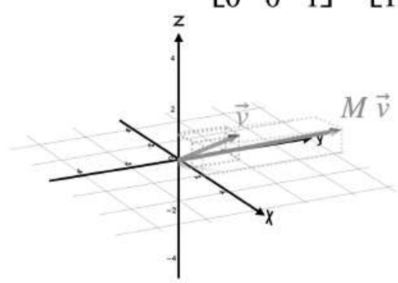
Watch video about [Algebraic Geometry]
Algebraic Geometry is Impossible Without These 6 Things

7. Linear Algebra

This branch studies vector spaces and the linear mappings between them. It focuses on understanding systems of linear equations, matrix operations, determinants, eigenvalues, and eigenvectors. Using these, linear algebra shows the structure of spaces and the transformations that preserve their linear properties.

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M\vec{v} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$



See book on [Linear Algebra]

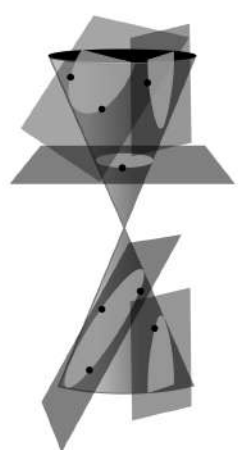
Watch video about [Linear Algebra]

The Core of Linear Algebra or Linear Algebra is Impossible Without These 8 Things

8. Arithmetic Geometry

The subject mostly deals with Diophantine problems, which are equations seeking integer or rational solutions, and explores how these can be understood through geometric principles. Arithmetic Geometry makes use of advanced algebraic concepts such as schemes.

$$x^2 + y^2 = 7z^2$$

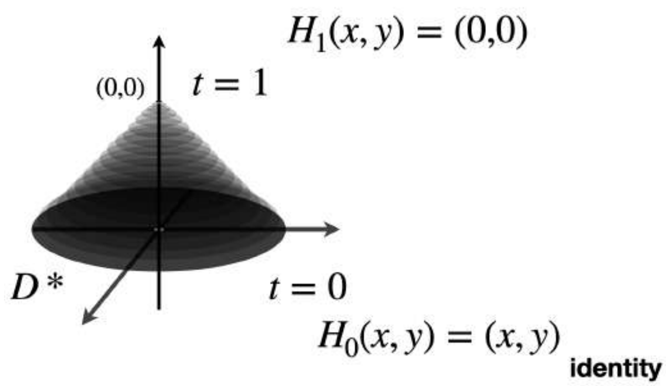


See book on [Arithmetic Geometry]()

Watch video about [Arithmetic Geometry]() The Core of Arithmetic Geometry

9. **Algebraic Topology**

It studies spaces through algebraic structures known as topological invariants, such as homotopy and homology groups. The goal is to assign algebraic structures such as groups, rings, and vector spaces to topological spaces. This helps to classify and distinguish these spaces based on their topological properties.

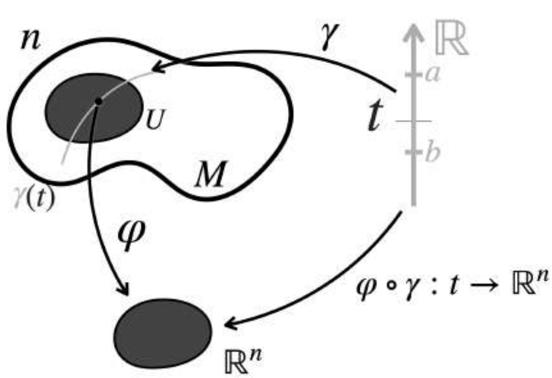


See book on [Algebraic Topology]()

10. **Differential Topology**

It's a field that focuses on studying properties of geometric structures that are preserved under smooth transformations, specifically within the context of manifolds.

The essence of differential topology involves looking at the local properties of manifolds by using calculus. By ensuring that the transition functions between local coordinate charts are smooth (or in other words, infinitely differentiable), differential topology allows for the application of differential and integral calculus on these manifolds.



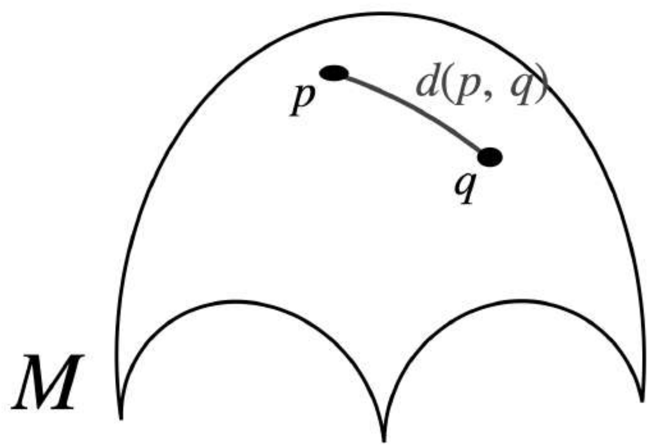
See book on [Differential Topology]

Watch videos about [Differential Topology]

How to Get to Manifolds Naturally or How to do Calculus on an Abstract Manifold

11. Differential Geometry

Differential geometry is the study of smooth shapes and structures. It uses calculus and linear algebra to explore properties of curves, surfaces, and higher-dimensional manifolds. Key topics include curvature, geodesics, and connections, with strong links to topology, analysis, and even theoretical physics.



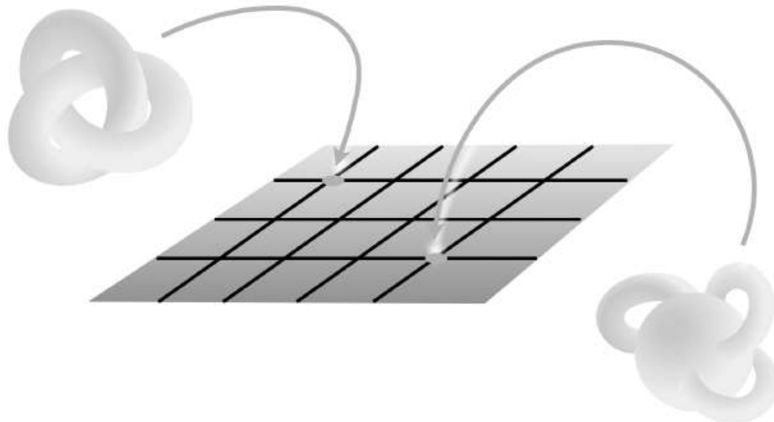
See book on [Differential Geometry]

Watch videos about [Differential Geometry]

The Core of Differential Geometry or
Differential Geometry is Impossible Without These 7 Things or
What are Tangent Spaces in Differential Geometry? or
How to get to Geodesics Naturally

12. Moduli Spaces

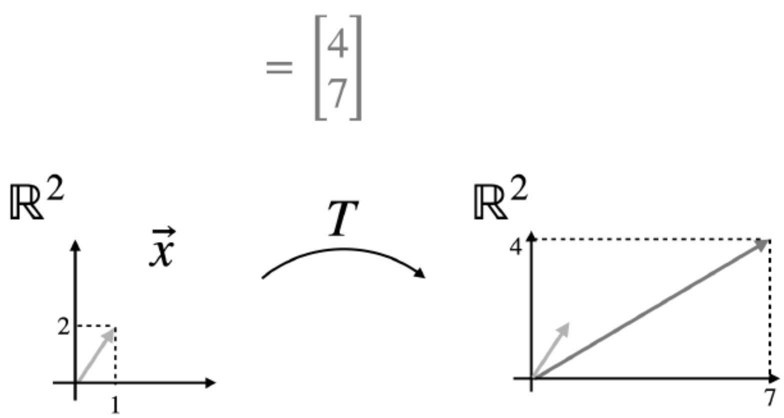
Moduli spaces allow for the geometric classification of objects like curves, surfaces, and more abstract structures by treating them as points in a geometric space. The approach of moduli spaces simplifies our understanding of these objects and makes their manipulation much easier. For example, moduli spaces of curves help in exploring how complex structures, such as Riemann surfaces, vary and interact under continuous deformations.



See book on Moduli Spaces

13. **Representation Theory**

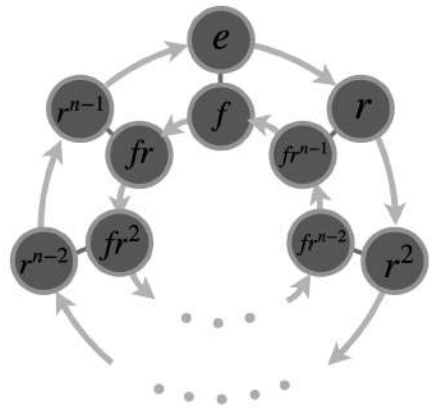
A branch that studies how groups, as sets of symmetries, can be represented through linear transformations on vector spaces. The main idea in representation theory is to associate each element of a group with a matrix such that the group's operation corresponds to matrix multiplication. This allows complex group structures to be studied using linear algebra techniques. A really important aspect of representation theory involves the study of how these matrices act on vector spaces. This reveals quite a bit about the group's structure through the properties of these actions, like whether they preserve the vector space structure.



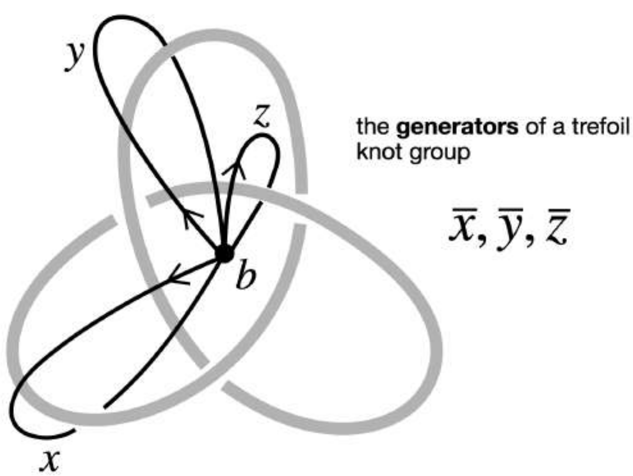
See book on Representation Theory

14. **Geometric and Combinatorial Group Theory**

The field is split into two main subfields: geometric group theory and combinatorial group theory. Geometric group theory focuses on the relationships between groups and geometric or topological structures, often studying groups through their actions on various spaces. It seeks to develop insights that benefit both the study of geometry or topology and group theory itself.



Combinatorial group theory, traditionally concerned with the study of groups presented by generators and relations, examines how groups are built and their properties deduced from these presentations. It first appeared from the study of discrete groups of isometries and the fundamental group concept. Group Theory has some strong connections with Representation Theory.

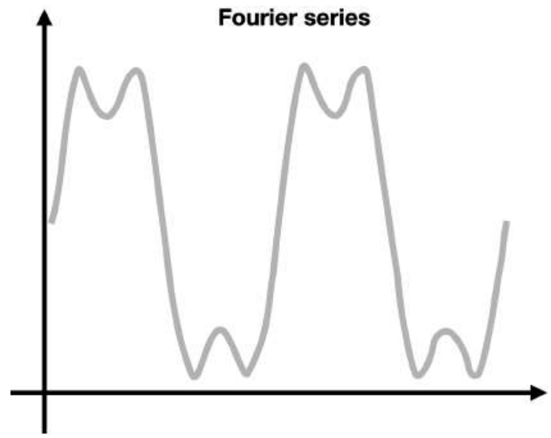


See book on [Geometric and Combinatorial Group Theory]

Watch videos about [Group Theory]
How We Got to the Classification of Finite Groups | Group Theory or
How to Visualize These 5 Fundamental Groups or
Example of an Interesting Lie Group: SE(2)

15. Fourier Analysis

Fourier analysis mostly focuses on how functions or signals can be expressed as sums or integrals of basic, oscillatory functions—like sine and cosine waves or, more generally, complex exponentials. Basically, it's all about understanding and manipulating functions through their frequency components.

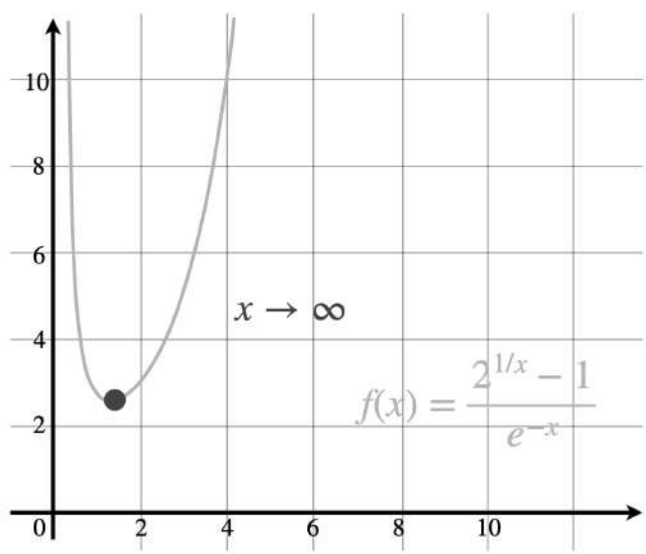


See book on [Fourier Analysis]

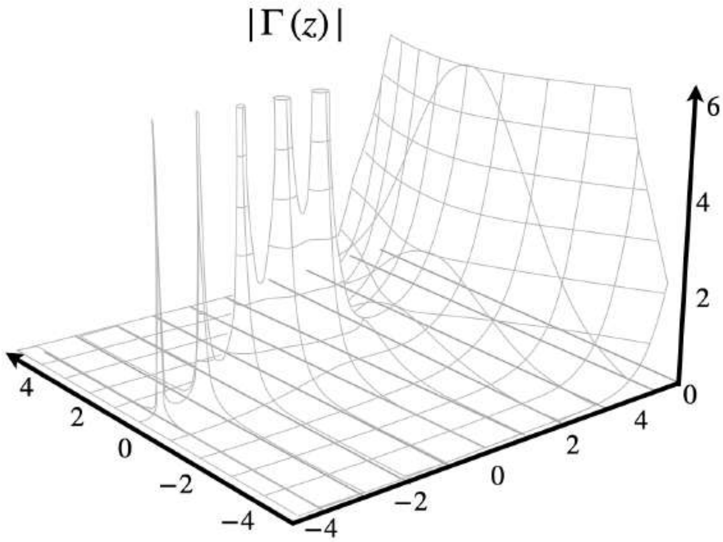
Watch video about [Fourier Series] Fourier Series is Impossible Without These 7 Things

16. Real Analysis and Complex Analysis

Real analysis studies real numbers and their functions, going into limits, continuity, integration, differentiation, and series to understand the foundations of calculus.



Complex analysis extends these ideas further to functions of complex numbers. It studies properties such as analyticity, contour integration, residues, and conformal mappings.



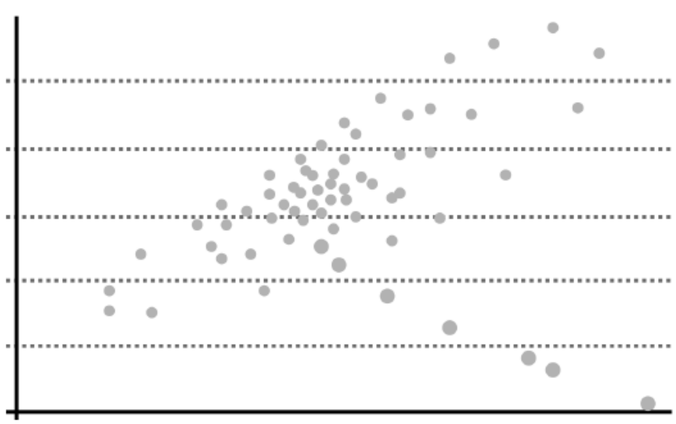
See book on [Real and Complex Analysis]

Watch video on Real and Complex [Analysis]

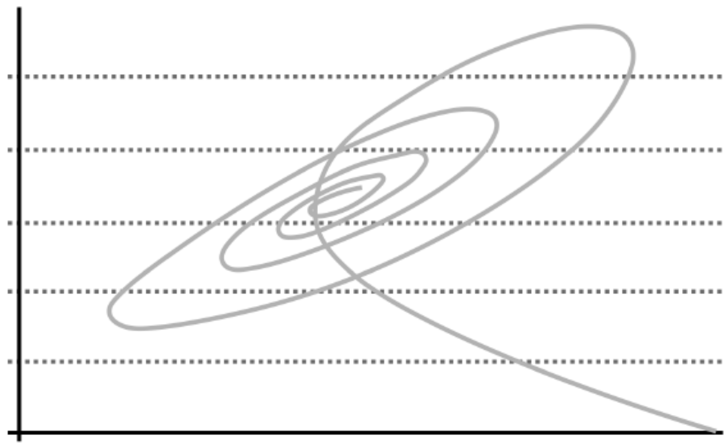
How to Learn Analysis Effectively or Is The Imaginary Unit Actually Equal to 1? or I Calculated the n-th Root of the Imaginary Unit and Look What I Found

17. Dynamics

Dynamics, or dynamical systems is basically the study of how systems evolve over time. This includes both discrete and continuous systems. Discrete dynamical systems operate in jumps over distinct, separate points in time, such as the steps in a numerical computation.



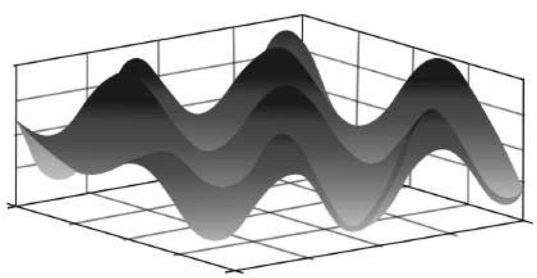
Continuous dynamical systems deal with changes occurring over continuous time, which resembles natural processes such as motion under gravitational forces.



See book on [Dynamical Systems]

18. Partial Differential Equations

(or PDEs) distinguish themselves by the presence of partial derivatives, indicating the rate of change of functions relative to multiple independent variables. This class includes a wide spectrum of equations, each of which models entirely different scenarios. For example, the heat equation models the distribution and flow of heat in a medium over time, while the wave equation describes the propagation of waves, such as sound or light waves, through space.

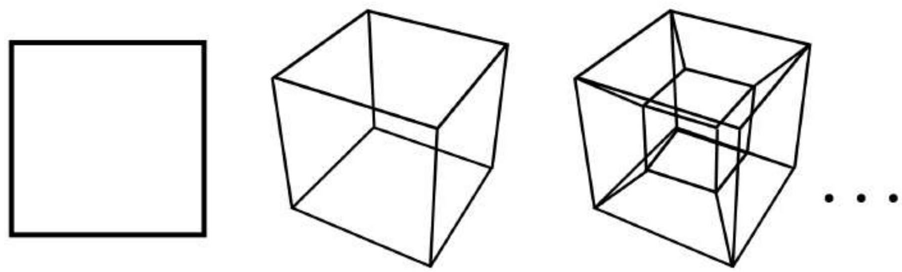


$$\frac{\partial^2 f}{\partial x \partial y}(x, y)$$

See book on [Partial Differential Equations]

19. Functional Analysis

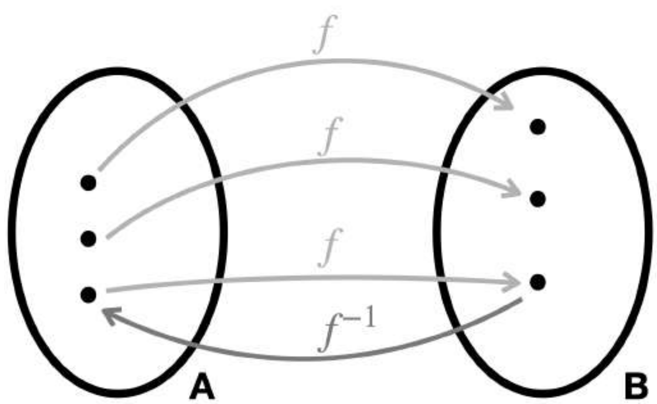
Functional analysis studies infinite-dimensional vector spaces—most notably Banach and Hilbert spaces—and the linear operators acting on them. It combines techniques to investigate convergence, continuity, spectral properties, and duality. These are applied across different areas such as quantum mechanics.



See book on Functional Analysis

20. Enumerative and Algebraic Combinatorics

Enumerative combinatorics is one of the oldest branches of mathematics, focusing on the straightforward task of counting configurations or structures. This is done by establishing bijective correspondences that simplify the counting process. Algebraic combinatorics, which is a more modern development, uses algebraic structures like groups and rings to solve combinatorial problems.

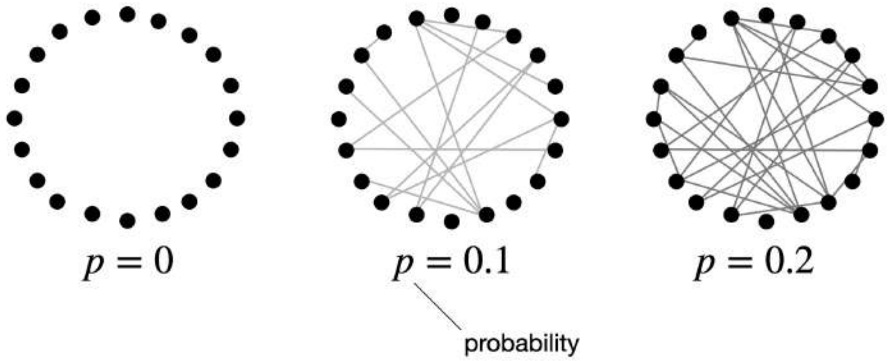


See book on Enumerative and Algebraic Combinatorics

Watch video about Combinatorics Mapping Combinatorics

21. Extremal and Probabilistic Combinatorics

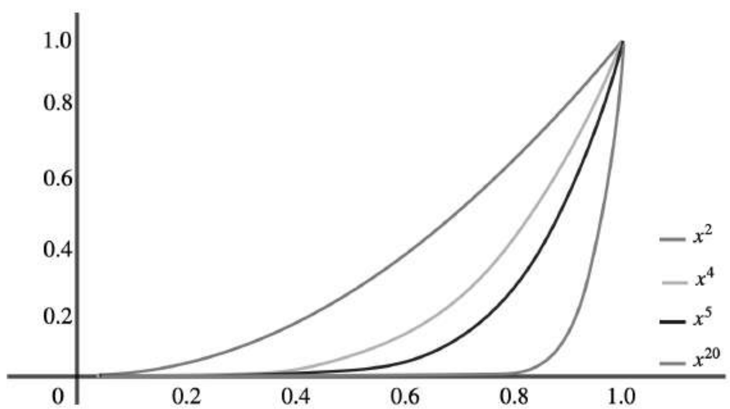
The foundational technique here, the probabilistic method, involves showing the existence of a combinatorial structure that satisfies certain properties by showing that the probability of randomly selecting such a structure is non-zero.



See book on Extremal and Probabilistic Combinatorics

22. Numerical Analysis

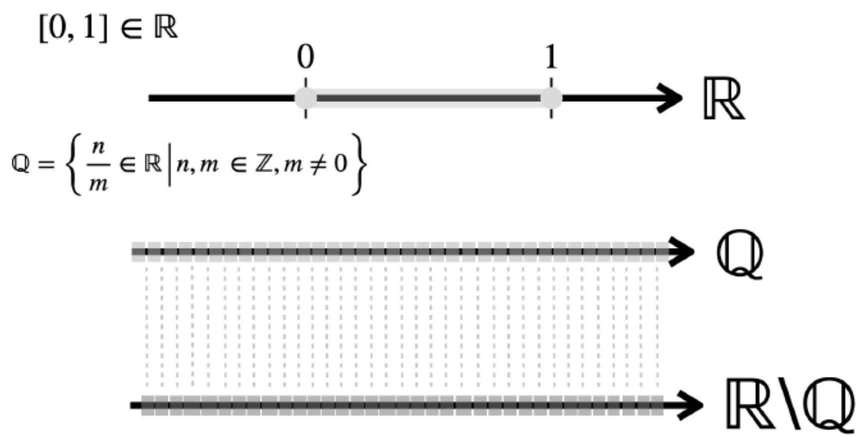
Numerical analysis deals with the development and implementation of numerical methods to solve mathematical problems. It deals with key concepts such as error analysis, stability, and convergence. These are really important in evaluating the effectiveness of numerical algorithms.



See book on Numerical Analysis

23. Set Theory

The study within this field focuses on understanding how sets operate, interact, and can be characterized in terms of their size and structure. This includes the study of both finite and infinite sets, with special attention given to infinite sets. It led us to the development of concepts like cardinality, which measures the "size" or number of elements in a set.



Plus, set theory explores the properties and implications of different axiomatic systems, such as the Zermelo-Fraenkel axioms. These axioms are foundational for all of mathematics, and they define the rules by which sets can be constructed and manipulated and include the Axiom of Choice. It is a very controversial and critical axiom, and it has implications in various fields of mathematics.

See book on Set Theory

24. Model Theory

Logic and model theory use formal language in order to define and study mathematical structures. It allows us to analyze how these structures follow axioms and rules. The study emphasizes the relationship between syntactic approaches (which are based on the formal languages and rules) and semantic views (which focus on meaning and truth within models).

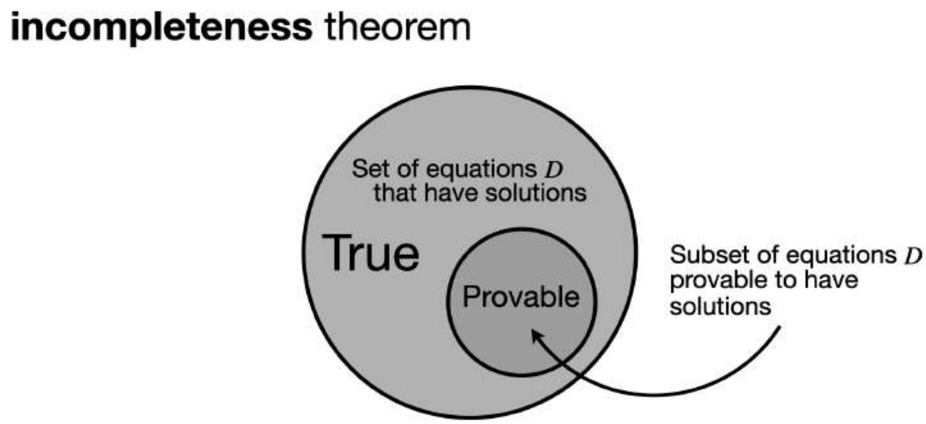
Model theory addresses results such as the completeness and incompleteness theorems. The completeness theorem tells us that if a statement is true in all models of a theory, then it can be proven within that theory.

completeness theorem if $\vDash P$, then $\vdash P$.

$$\overline{\vdash P \Rightarrow (Q \Rightarrow R)} \quad \overline{\vdash \left(P \Rightarrow (Q \Rightarrow Q')\right) \Rightarrow \left((P \Rightarrow Q) \Rightarrow (P \Rightarrow Q')\right)} \quad \overline{\vdash \neg\neg P \Rightarrow P}$$

$$\frac{\vdash P \quad \vdash P \implies Q}{\vdash Q}$$

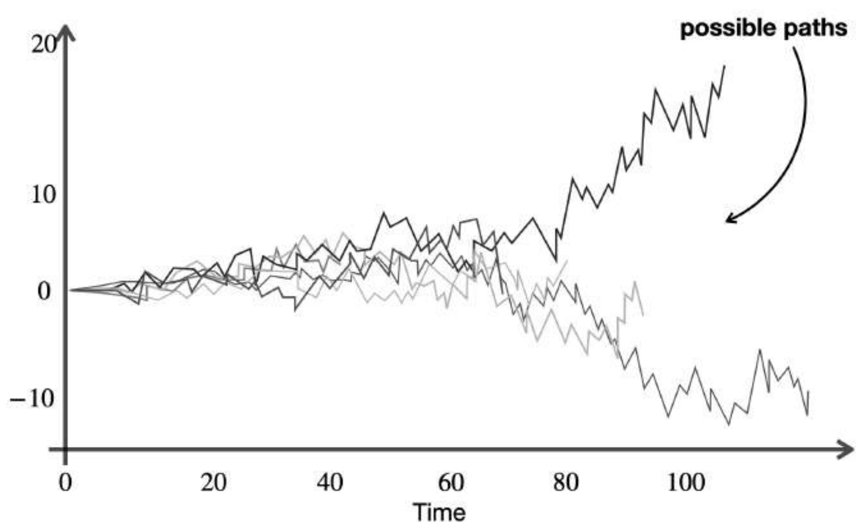
The incompleteness theorem, though, shows the intrinsic limitations of-systems. It demonstrates that no consistent set of axioms can prove all truths about natural numbers.



See book on [Model Theory]

25. Stochastic Processes

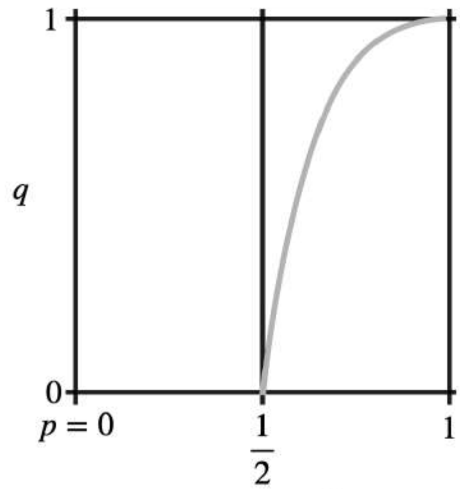
The branch examines models that describe the evolution of systems influenced by random variables over time. This field is super important in understanding phenomena where outcomes are inherently uncertain and can evolve in complex, and often non-deterministic ways.



See book on [Stochastic Processes]

26. Probabilistic Models of Critical Phenomena

This field mostly uses probabilistic methods to model and analyze phenomena that involve uncertainty and randomness. Specifically, it focuses on transitions and changes that occur at critical points.



The survival probability θ versus p

$$\theta(p)\begin{cases} 0 \text{ if } p \le \frac{1}{2}, \\ \frac{1}{p^2}(2p-1), \text{ if } p \ge \frac{1}{2} \end{cases}$$

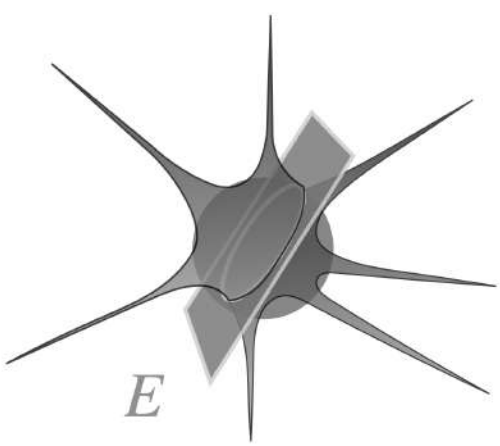
$$p = p_c = \frac{1}{2} \longrightarrow \text{critical value}$$

See book on [Probabilistic Models of Critical Phenomena]

27. High-Dimensional Geometry and Its Probabilistic Analogues

The field studies-geometric objects in spaces of high dimension, where objects that appear different in lower dimensions can show similar probabilistic properties.

It introduces the isoperimetric principle, which in high dimensions has interesting connections with probabilistic measures. This suggests that geometric and probabilistic properties converge in higher dimensions.

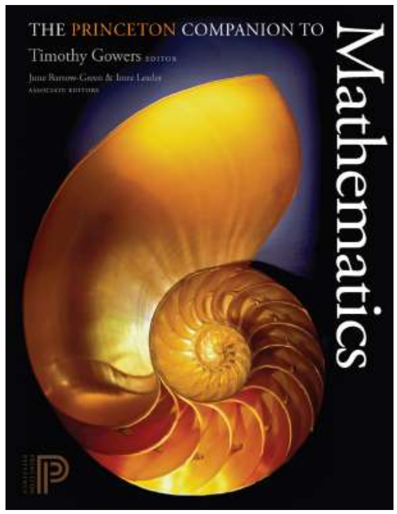


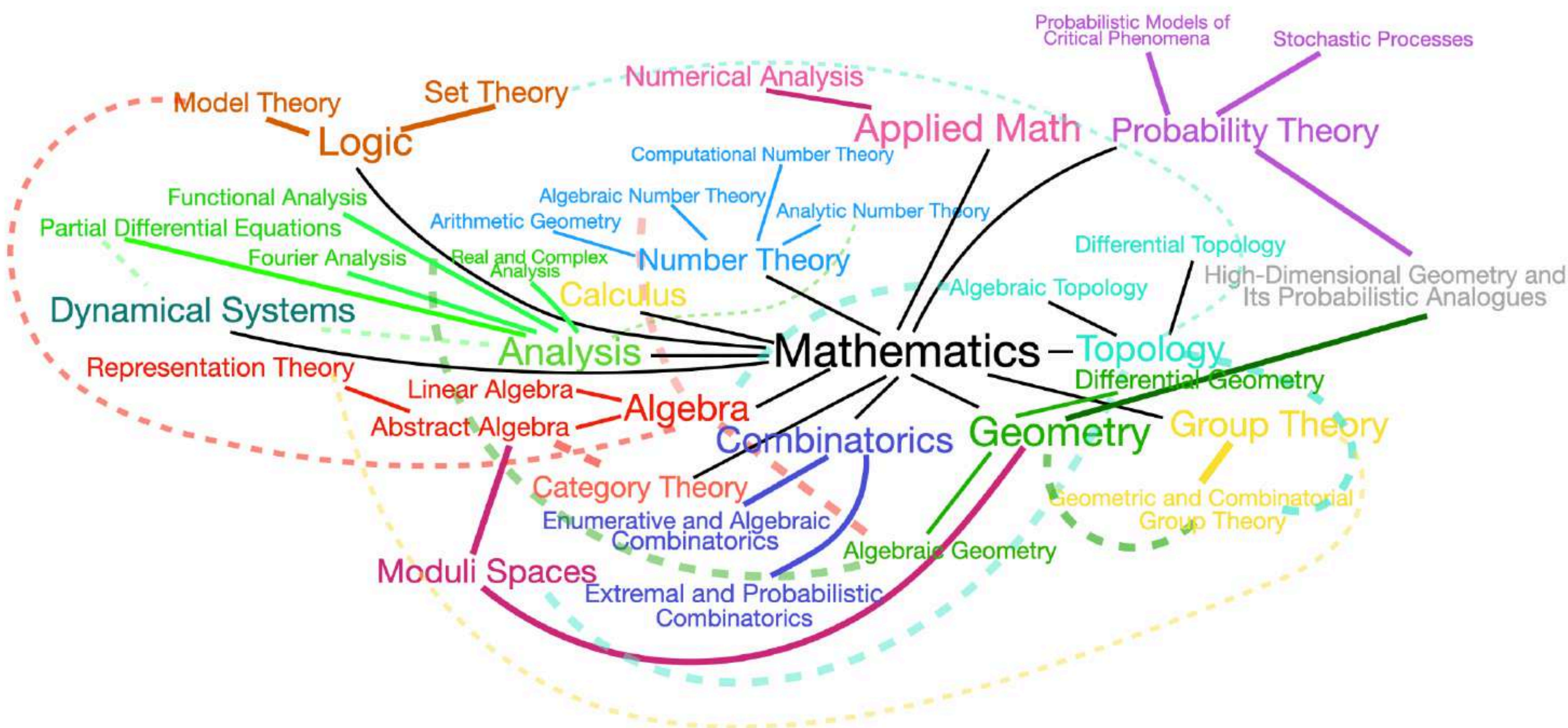
the intersection of a set T with a random subspace E

See book on [High-Dimensional Geometry and Its Probabilistic Analogues]

This list could be infinite, it's impossible to actually map all of mathematics, but as a general layout, it's pretty comprehensive. Mind that we're focused mostly on pure mathematics rather than applications, like physics, computer science and so on. We're sure there's more that can be added, but hopefully this gives you a pretty good idea of what kind of foundational branches there are. Let us know in the comments what other fields should be included.

This video was based on the [Princeton Companion to Mathematics](https://press.princeton.edu/titles/8350.html).





Probabilistic Models of Critical Phenomena
Stochastic Processes
Numerical Analysis
Model Theory
Set Theory
Logic
Applied Math
Probability Theory
Computational Number Theory
Algebraic Number Theory
Analytic Number Theory
Functional Analysis
Arithmetic Geometry
Partial Differential Equations
Fourier Analysis
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Topology
Representation Theory
Differential Geometry
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Algebraic Geometry
Moduli Spaces
Extremal and Probabilistic Combinatorics