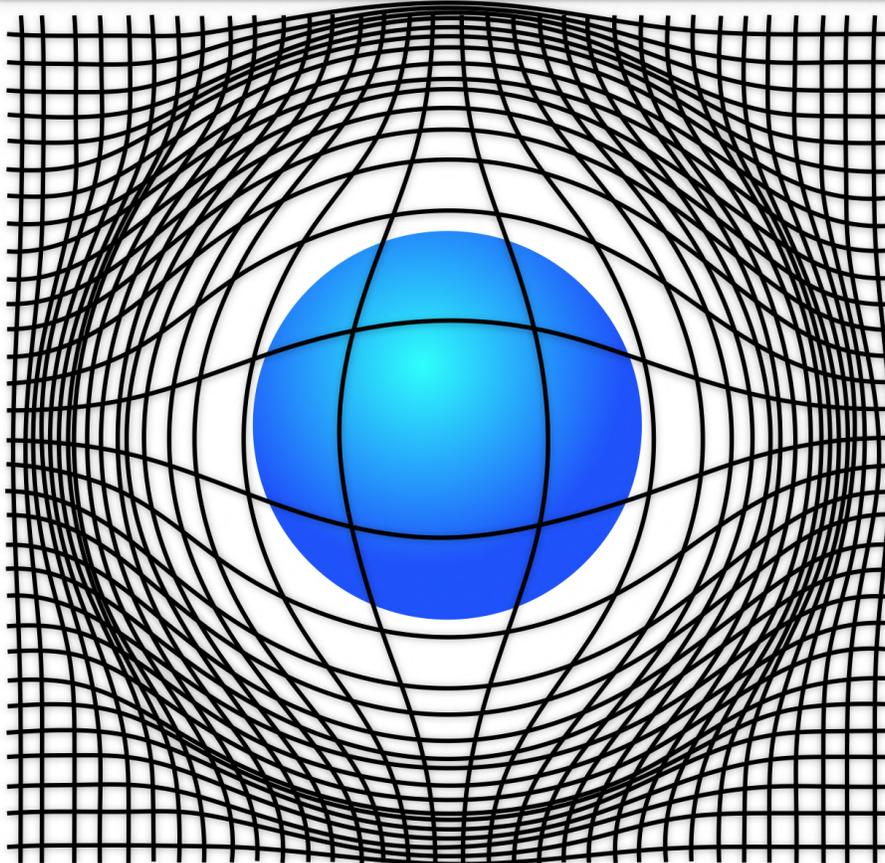# How Many Solutions to Einstein's Equations Are There?

by DiBeos

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4}T_{\mu\nu}$$

"Einstein's general theory of relativity is the greatest feat of human thinking about nature, the most amazing combination of philosophical penetration, physical intuition and mathematical skill." – **Max Born**
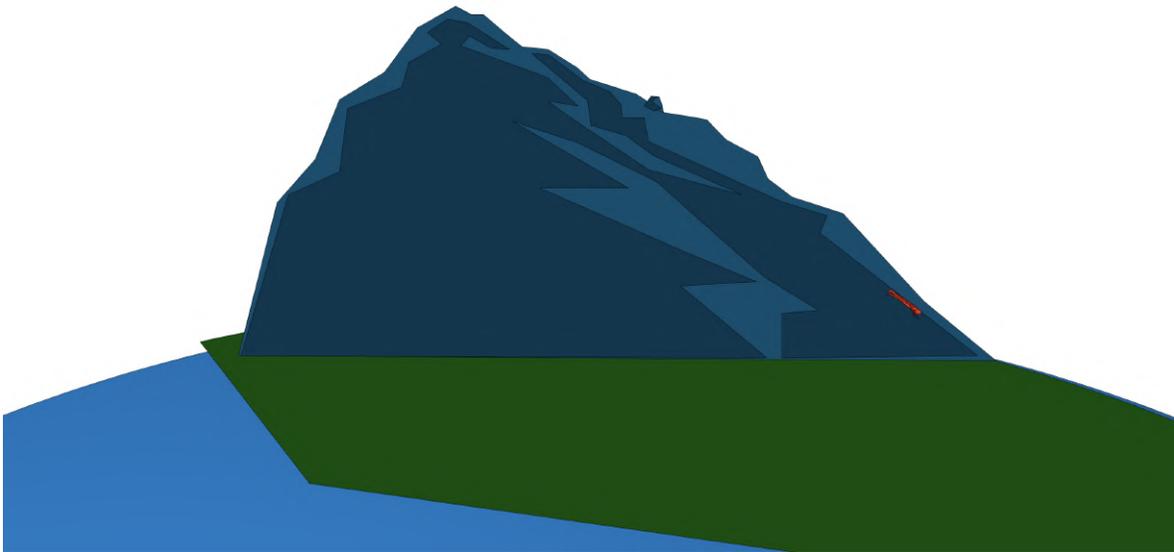
Introduction

Let's begin with something surprisingly simple. You're walking down a hill.

Gravity pulls you down, you stumble and fall on your face! Sorry about that.
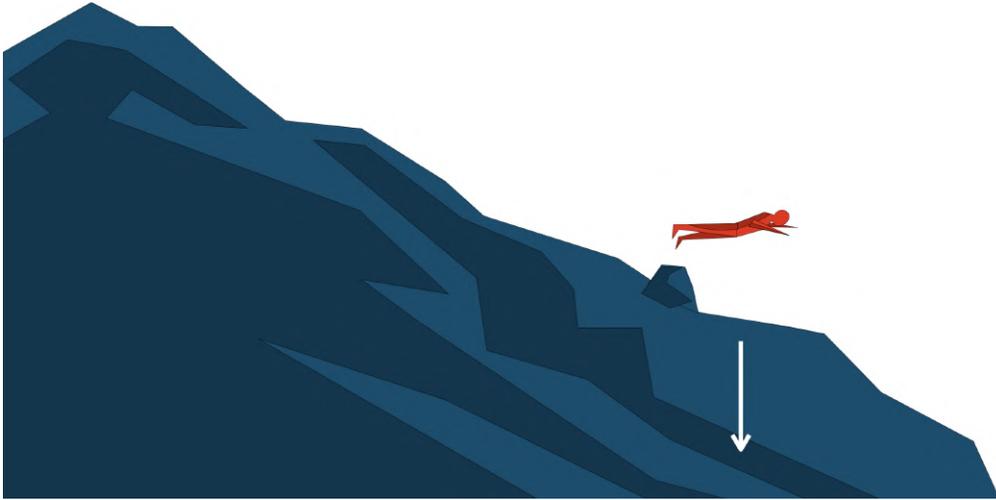
Right after that, your body will definitely obey the geometry of the hill, and ultimately of the Earth itself.

Einstein's equations describe how space and time themselves curve in response to matter and energy.

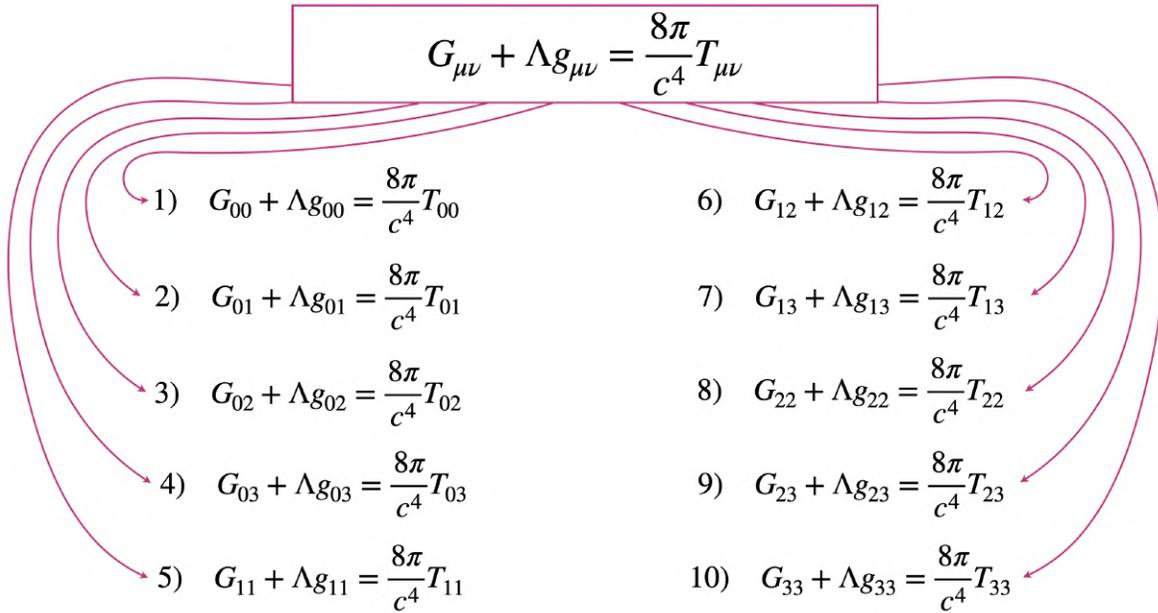$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4} T_{\mu\nu}$$

But here's the weird thing: even if we remove all matter and energy, these equations still have solutions. Theoretically, these equations could describe entire universes. Not just one, but infinitely many...
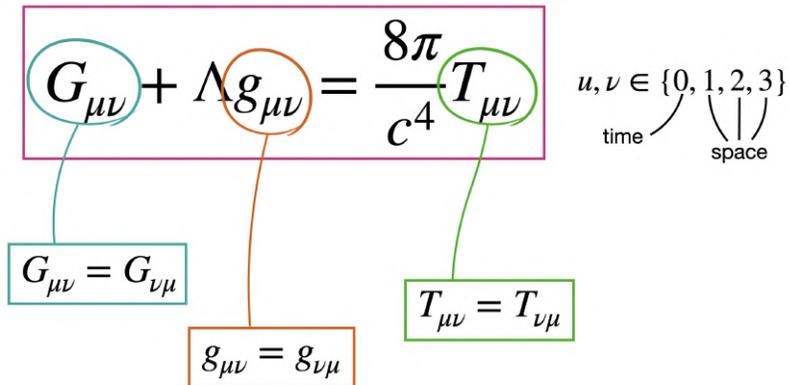
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4} T_{\mu\nu}$$

So, *how many solutions are there to Einstein's field equations?* The honest answer is: we don't know. But we do know this: the number is huge, probably **infinite**.

Einstein's equations are 10 interlinked nonlinear partial differential equations.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4}T_{\mu\nu}$$

1) $G_{00} + \Lambda g_{00} = \dfrac{8\pi}{c^4}T_{00}$

2) $G_{01} + \Lambda g_{01} = \dfrac{8\pi}{c^4}T_{01}$

3) $G_{02} + \Lambda g_{02} = \dfrac{8\pi}{c^4}T_{02}$

4) $G_{03} + \Lambda g_{03} = \dfrac{8\pi}{c^4}T_{03}$

5) $G_{11} + \Lambda g_{11} = \dfrac{8\pi}{c^4}T_{11}$

6) $G_{12} + \Lambda g_{12} = \dfrac{8\pi}{c^4}T_{12}$

7) $G_{13} + \Lambda g_{13} = \dfrac{8\pi}{c^4}T_{13}$

8) $G_{22} + \Lambda g_{22} = \dfrac{8\pi}{c^4}T_{22}$

9) $G_{23} + \Lambda g_{23} = \dfrac{8\pi}{c^4}T_{23}$

10) $G_{33} + \Lambda g_{33} = \dfrac{8\pi}{c^4}T_{33}$

Actually, there should be 16 of such equations, since each side has rank-2 symmetric tensors; meaning it has 2 indices μ and ν, and is symmetric under exchange:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4}T_{\mu\nu}$$

$u, \nu \in \{0, 1, 2, 3\}$

time   space

$G_{\mu\nu} = G_{\nu\mu}$

$g_{\mu\nu} = g_{\nu\mu}$

$T_{\mu\nu} = T_{\nu\mu}$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4} T_{\mu\nu}$$

1) $G_{00} + \Lambda g_{00} = \frac{8\pi}{c^4} T_{00}$

2) $G_{01} + \Lambda g_{01} = \frac{8\pi}{c^4} T_{01}$

3) $G_{02} + \Lambda g_{02} = \frac{8\pi}{c^4} T_{02}$

4) $G_{03} + \Lambda g_{03} = \frac{8\pi}{c^4} T_{03}$

$G_{10} + \Lambda g_{10} = \frac{8\pi}{c^4} T_{10}$

5) $G_{11} + \Lambda g_{11} = \frac{8\pi}{c^4} T_{11}$

6) $G_{12} + \Lambda g_{12} = \frac{8\pi}{c^4} T_{12}$

7) $G_{13} + \Lambda g_{13} = \frac{8\pi}{c^4} T_{13}$

$G_{20} + \Lambda g_{20} = \frac{8\pi}{c^4} T_{20}$

$G_{21} + \Lambda g_{21} = \frac{8\pi}{c^4} T_{21}$

8) $G_{22} + \Lambda g_{22} = \frac{8\pi}{c^4} T_{22}$

9) $G_{23} + \Lambda g_{23} = \frac{8\pi}{c^4} T_{23}$

$G_{30} + \Lambda g_{30} = \frac{8\pi}{c^4} T_{30}$

$G_{31} + \Lambda g_{31} = \frac{8\pi}{c^4} T_{31}$

$G_{32} + \Lambda g_{32} = \frac{8\pi}{c^4} T_{32}$

10) $G_{33} + \Lambda g_{33} = \frac{8\pi}{c^4} T_{33}$

But only 10 of such equations are independent, because of this symmetry, which implies that the components in each triangle below are the same. This general matrix may represent $G_{\mu\nu}$, $g_{\mu\nu}$ or $T_{\mu\nu}$.

$$G_{\mu\nu} = G_{\nu\mu} \qquad g_{\mu\nu} = g_{\nu\mu} \qquad T_{\mu\nu} = T_{\nu\mu}$$
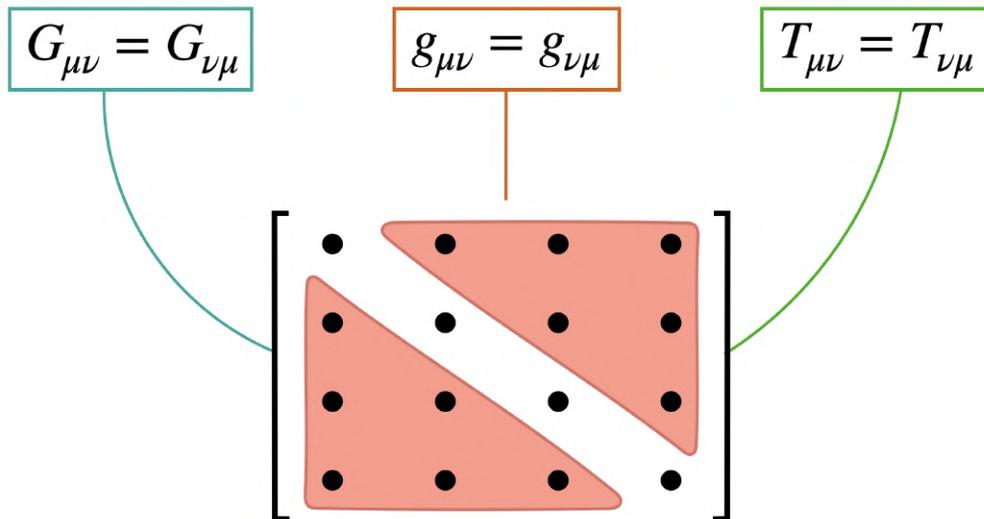
Therefore, all we need to do is count the number of components inside of the triangles plus in the diagonal rectangle:
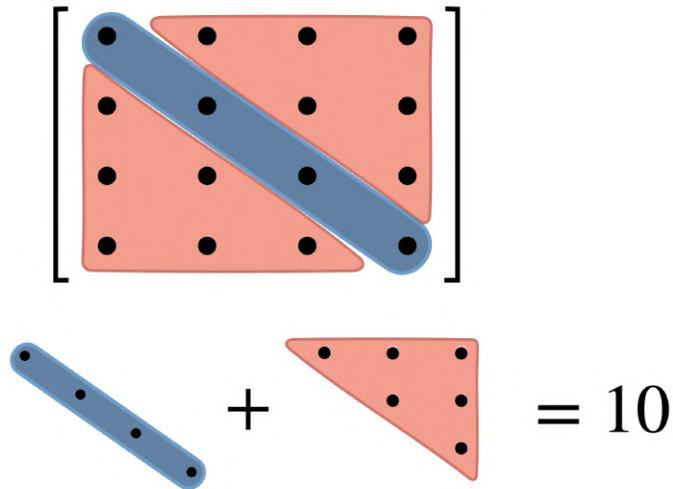
Again, these are the 10 independent equations:

1) $G_{00} + \Lambda g_{00} = \dfrac{8\pi}{c^4} T_{00}$ 6) $G_{12} + \Lambda g_{12} = \dfrac{8\pi}{c^4} T_{12}$

2) $G_{01} + \Lambda g_{01} = \dfrac{8\pi}{c^4} T_{01}$ 7) $G_{13} + \Lambda g_{13} = \dfrac{8\pi}{c^4} T_{13}$

3) $G_{02} + \Lambda g_{02} = \dfrac{8\pi}{c^4} T_{02}$ 8) $G_{22} + \Lambda g_{22} = \dfrac{8\pi}{c^4} T_{22}$

4) $G_{03} + \Lambda g_{03} = \dfrac{8\pi}{c^4} T_{03}$ 9) $G_{23} + \Lambda g_{23} = \dfrac{8\pi}{c^4} T_{23}$

5) $G_{11} + \Lambda g_{11} = \dfrac{8\pi}{c^4} T_{11}$ 10) $G_{33} + \Lambda g_{33} = \dfrac{8\pi}{c^4} T_{33}$

Let's see some concrete solutions.

Minkowski spacetime

In Cartesian coordinates $(x^0, x^1, x^2, x^3) = (t, x, y, z)$, the **Minkowski metric** is:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

(1) Minkowski Spacetime

$$\left(x^0, x^1, x^2, x^3\right) = \left(t, x, y, z\right)$$

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
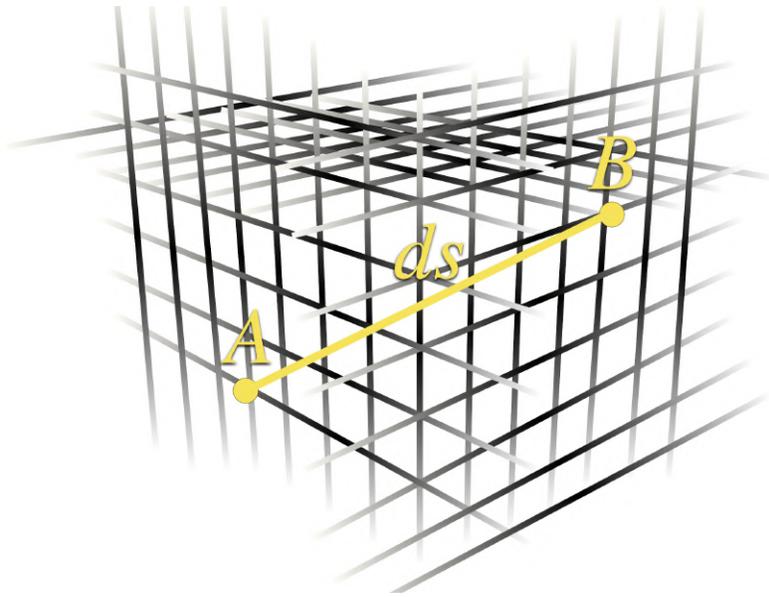
And so the line element is:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

$$ds^2 = -\, dt^2 + dx^2 + dy^2 + dz^2$$

This line element tells us how spacetime itself measures intervals between nearby events.

As you can see, this metric is flat, meaning that all components of the *Riemann curvature tensor* vanish:
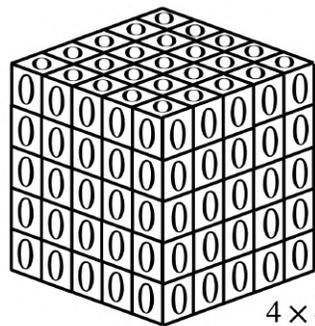
$$R^{\rho}_{\sigma\mu\nu} = 0$$

Which implies that the *Einstein tensor* is also zero everywhere.

$$R^{\rho}_{\sigma\mu\nu} = 0$$

$$\Downarrow$$

$$G_{\mu\nu} = 0 \quad = \overset{0}{\cancel{R}}_{\mu\nu} - \frac{1}{2}\overset{0}{\cancel{R}}g_{\mu\nu}$$

$4 \times 4 \times 4 \times 4$

This is a vacuum solution of Einstein's equations with $\Lambda = 0$, i.e. the *cosmological constant*, responsible for what we call **dark energy**, is set to zero everywhere.
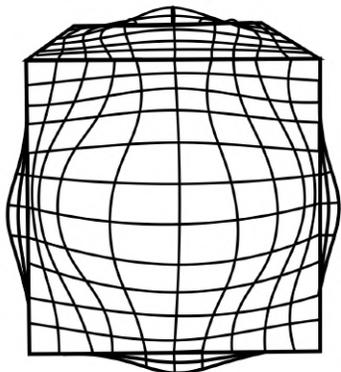
$$G_{\mu\nu}^{\;\;\;0} + \Lambda^{\;0} g_{\mu\nu} = \frac{8\pi}{c^4} T_{\mu\nu} \qquad \textbf{vacuum solution}$$

cosmological constant
(dark energy)

Schwarzschild spacetime

In spherical coordinates, the **Schwarzschild metric** is this:

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2GM}{rc^2}\right) c^2 & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

gravitational constant   total mass of body   speed of light

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2GM}{rc^2}\right) c^2 & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$
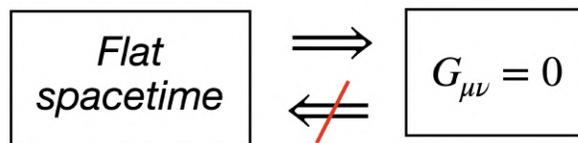
It describes how a massive object with spherical symmetry (like a planet or a star) bends spacetime around it, and inside of it as well, as we will see in a moment.
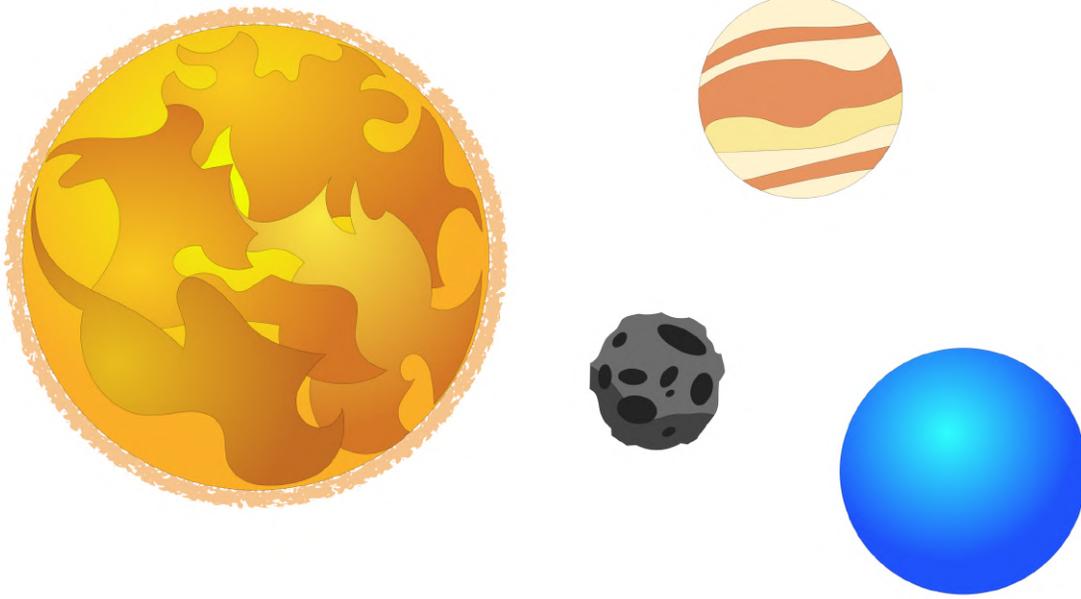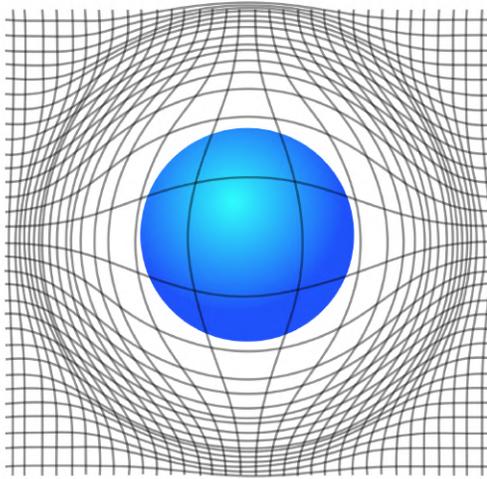
The line element here is:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2$$

This is also a vacuum solution $\left(T_{\mu\nu} = 0\right)$, and as a consequence $G_{\mu\nu} = 0$. However, it does not mean that curvature is zero. Indeed:

In our case, we are describing what happens outside the object responsible for spacetime curvature. That's why $T_{\mu\nu} = 0$, but curvature itself is not zero.

$$T_{\mu\nu} = 0 \quad \text{matter energy momentum}$$

$$R^{\rho}_{\ \sigma\mu\nu} \neq 0 \quad \text{curvature}$$

Ok, but what about inside the massive body? Inside, we cannot assume $T_{\mu\nu} = 0$. Instead, Einstein's equations are often solved by approximating the interior of the star, or planet, as a **perfect fluid**:

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu}$$

perfect fluid

This specific *stress-energy tensor* produces this Schwarzschild **interior** solution: ($R :=$ radius of the body)

$$g_{\mu\nu} = \begin{pmatrix} -\frac{1}{4}\left(3\sqrt{1-\frac{2GM}{Rc^2}} - \sqrt{1-\frac{2GMr^2}{R^3c^2}}\right)^2 c^2 & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GMr^2}{R^3c^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

For which the line element is:

$$ds^2 = -\frac{1}{4}\left(3\sqrt{1-\frac{2GM}{Rc^2}} - \sqrt{1-\frac{2GMr^2}{R^3c^2}}\right)^2 c^2 dt^2 + \left(1-\frac{2GMr^2}{R^3c^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

> **Note**: *If you'd like to be the first to find out when we launch our very first books and courses, sign up with your email address on our homepage, dibeos.net.*

There are 3 important things to notice before moving on:

1. **Point at infinity:** $(r \to \infty)$

Let's pick a point at infinity and see what happens with the Schwarzschild metric there.
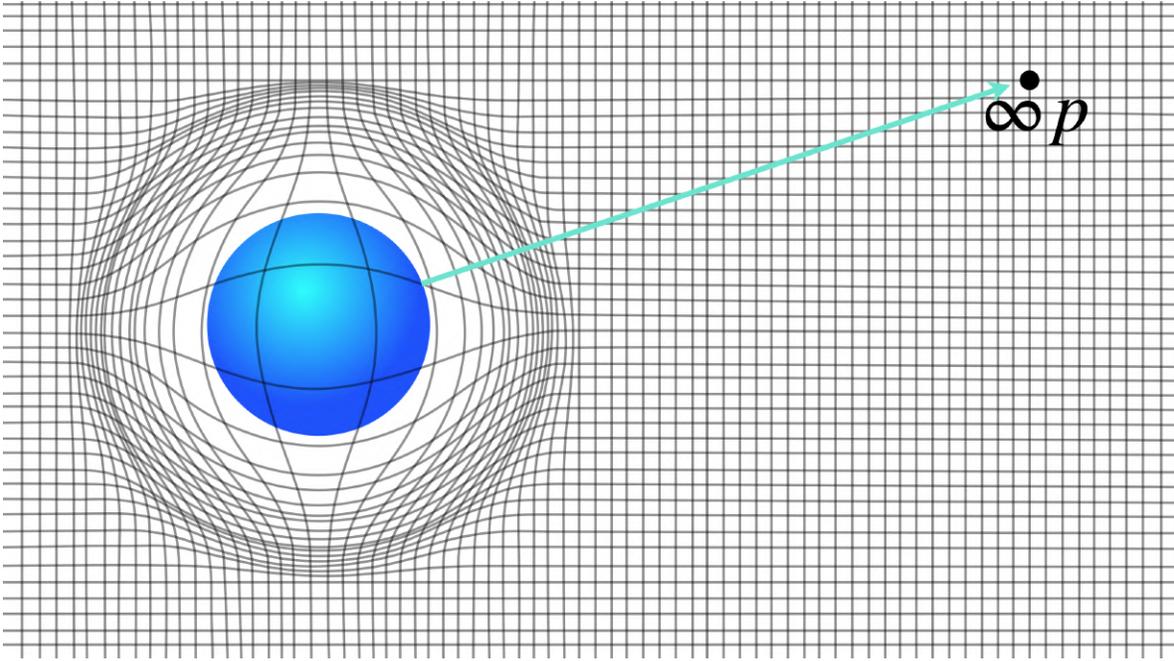
As we've seen up to this point, there are 2 possible Schwarzschild metrics: one for points in its **interior**, and the other for points in its **exterior**.

Since we want to see what happens to the metric at a point at infinity (and we'll see what that means in a moment), then we clearly must use the Schwarzschild exterior metric: $(r \to \infty)$

$$(r \to \infty)$$

$$
g_{\mu\nu} = \begin{pmatrix}
-\left(1 - \cancel{\frac{2GM}{rc^2}}^{\,0}\right)c^2 & 0 & 0 & 0 \\
0 & \left(1 - \cancel{\frac{2GM}{rc^2}}^{\,0}\right)^{-1} & 0 & 0 \\
0 & 0 & \cancel{r^2}^{\,\infty} & 0 \\
0 & 0 & 0 & \cancel{r^2 \sin^2\theta}^{\,\infty}
\end{pmatrix}
$$

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$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \infty & 0 \\ 0 & 0 & 0 & \infty \end{pmatrix} \quad , \quad c \equiv 1.$$

I mean, look at the time ($g_{00} = -1$) and radial ($g_{11} = 1$) entries, which are identical to the ones for the Minkowski metric. It does make sense, since if you move infinitely far from a massive object its gravitational influence tends to zero, i.e. you find yourself (practically speaking) in a flat portion of spacetime. More technically, we'd say that *"At infinity, spacetime is asymptotically flat: locally Minkowski in the t and r directions/components"*.

$$(r \to \infty)$$

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \cancelto{0}{\frac{2GM}{rc^2}}\right)c^2 & 0 & 0 & 0 \\ 0 & \left(1 - \cancelto{0}{\frac{2GM}{rc^2}}\right)^{-1} & 0 & 0 \\ 0 & 0 & \cancelto{\infty}{r^2} & 0 \\ 0 & 0 & 0 & \cancelto{\infty}{r^2 \sin^2\theta} \end{pmatrix}$$

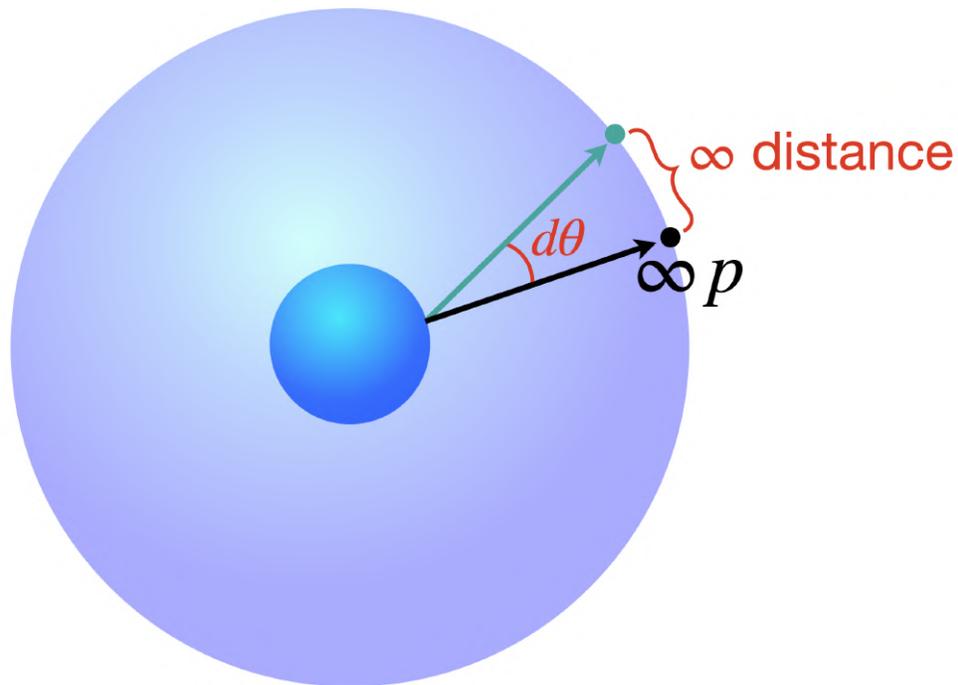Ok, but what about those ugly "∞" symbols in the angular (g_{22} and g_{33}) entries?

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \infty & 0 \\ 0 & 0 & 0 & \infty \end{pmatrix}$$

angular

$\infty = g_{22}$

$= g_{33}$

Well, as $r \to \infty$, the sphere is infinitely large. So the metric components for angular directions diverge, simply reflecting the fact that moving a tiny angle at infinity sweeps an infinite physical distance.


∞ distance

$d\theta$

$\infty\, p$

In the limit, all directions collapse to the same ideal point at infinity, similar to what happens in **Projective Geometry**. Don't worry if you have never studied projective geometry. The intuition is really simple, but powerful:

In projective geometry, you can do something called *compactification* of the Euclidean plane by adding "points at infinity".
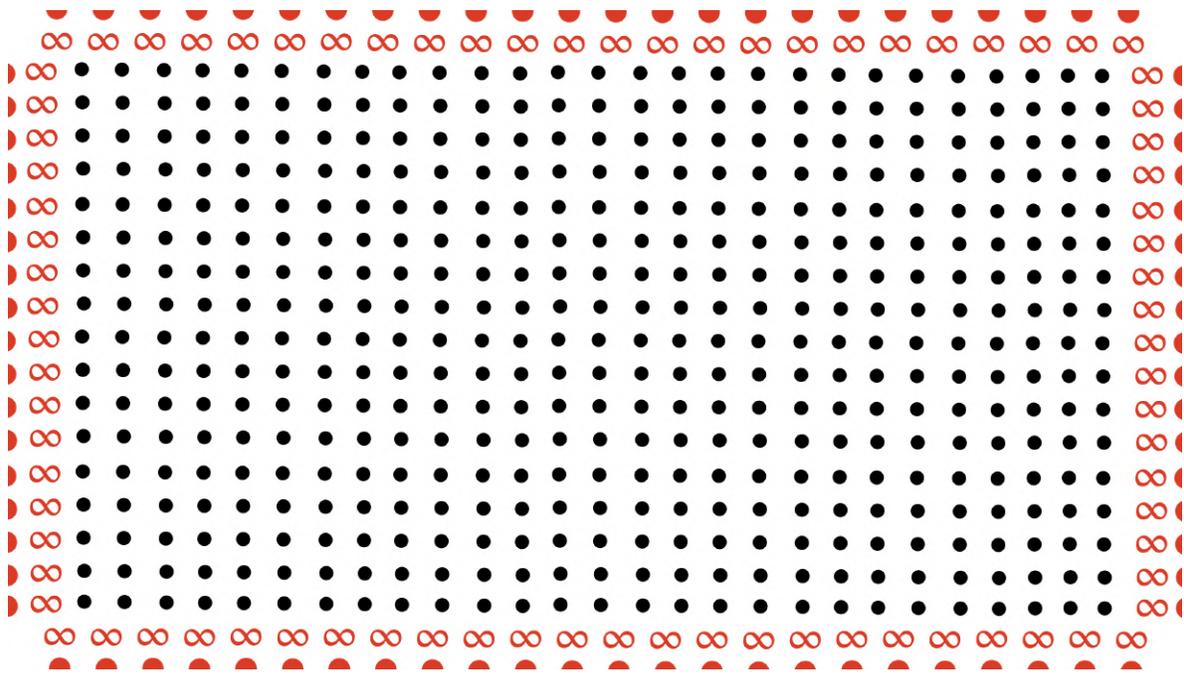
Each family of parallel lines meets at a unique point at infinity, and the collection of such points forms the *line at infinity*.
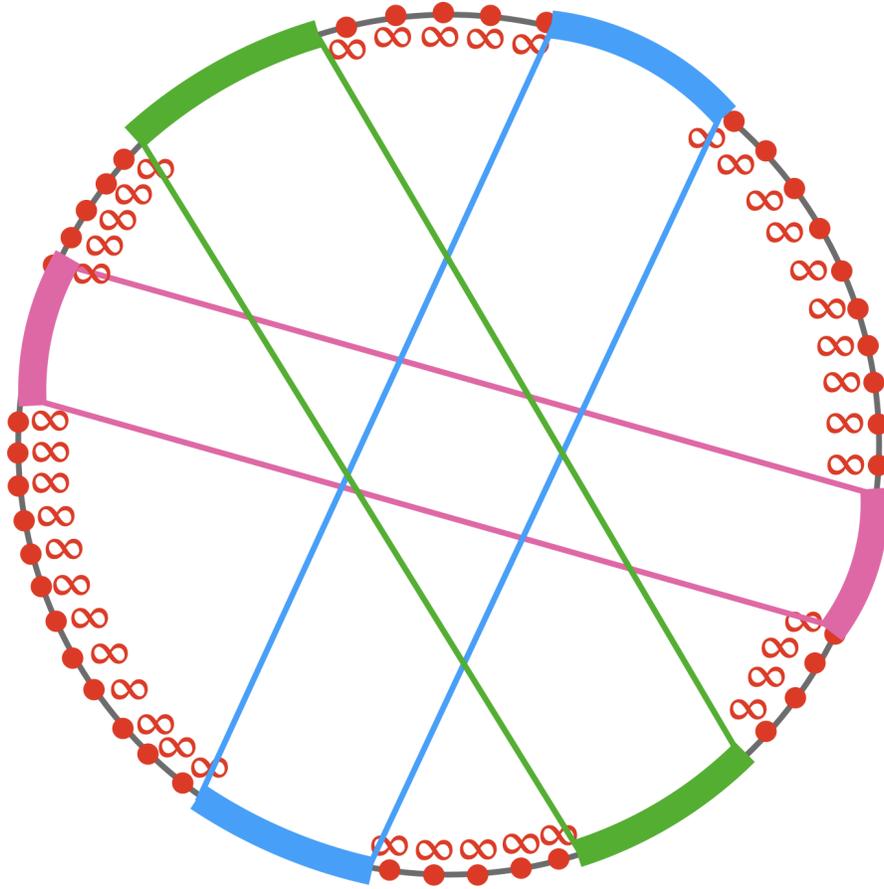
Of course, this line looks more like a circle in our drawing here, because we can't really represent on screen a line at infinity, but I hope you get the idea.

Anyway, the correct (and precise) Minkowski metric that you get (written in spherical coordinates) after moving infinitely away from the massive body is:

$$g_{\mu\nu} \xrightarrow{r \to \infty} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

2. **The metric right on the surface:** $(r = R)$

What happens if we take the interior Schwarzschild metric and study it for a point *on the surface* of the spherical body $(r = R)$?

Well, let's see:

$$g_{\mu\nu}\Big|_{r=R} = \begin{pmatrix} -\dfrac{1}{4}\left(3\sqrt{1-\dfrac{2GM}{Rc^2}} - \sqrt{1-\dfrac{2GMR^2}{R^3c^2}}\right)^2 c^2 & 0 & 0 & 0 \\ 0 & \left(1-\dfrac{2GMR^2}{R^3c^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & R^2\sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} -\left(1-\dfrac{2GM}{Rc^2}\right)c^2 & 0 & 0 & 0 \\ 0 & \left(1-\dfrac{2GM}{Rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & R^2\sin^2\theta \end{pmatrix} \quad \begin{array}{c} \textbf{exterior} \\ \textbf{Schwarzschild} \\ \textbf{metric} \end{array}$$

Check this out! We just recovered the exterior Schwarzschild metric at a point $r = R$. I mean, it makes sense if you think about it...

3. Singularity: $(r = 0)$

Now we are ready for the coolest part of it! Let's start with the exterior metric (since it's the simplest one). And to make things more tangible, let us pick the planet Earth as our massive body (mass: $M_E \approx 5.9722 \cdot 10^{24}$ kg). Recall that $G \approx 6.6743 \cdot 10^{-11}\,m^3/kg \cdot s^2$ and $c \approx 2.9979 \cdot 10^8$ m/s.

First, we substitute these values:

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \dfrac{\overbrace{2GM_E}^{\approx\,0.0088702}}{rc^2}\right)c^2 & 0 & 0 & 0 \\ 0 & \left(1 - \dfrac{\overbrace{2GM_E}^{\approx\,0.0088702}}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

exterior metric

$$\Downarrow$$

$$g_{\mu\nu} \simeq \begin{pmatrix} -\left(1 - \dfrac{0.0088702}{r}\right)c^2 & 0 & 0 & 0 \\ 0 & \left(1 - \dfrac{0.0088702}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

Then, we can clearly see that if $r = \frac{2GM}{c^2} \approx 0.0088702$ m ≈ 8.87 mm, we get the following metric:

$$(*) \qquad g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 \\ 0 & 0 & \dfrac{4G^2M^2}{c^4} & 0 \\ 0 & 0 & 0 & \dfrac{4G^2M^2}{c^4}\sin^2\theta \end{pmatrix}$$

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Which is actually nonsense, because we just contradicted ourselves! Did you catch where exactly? Well, you don't need to know the exact radius of the Earth from the top of your head ($R_E \approx 6.371$ km) to notice that a point at the distance of $r \approx 8.87$ mm from the center is located inside our planet. And therefore, we cannot use the exterior Schwarzschild metric as we just did! This is a contradiction.
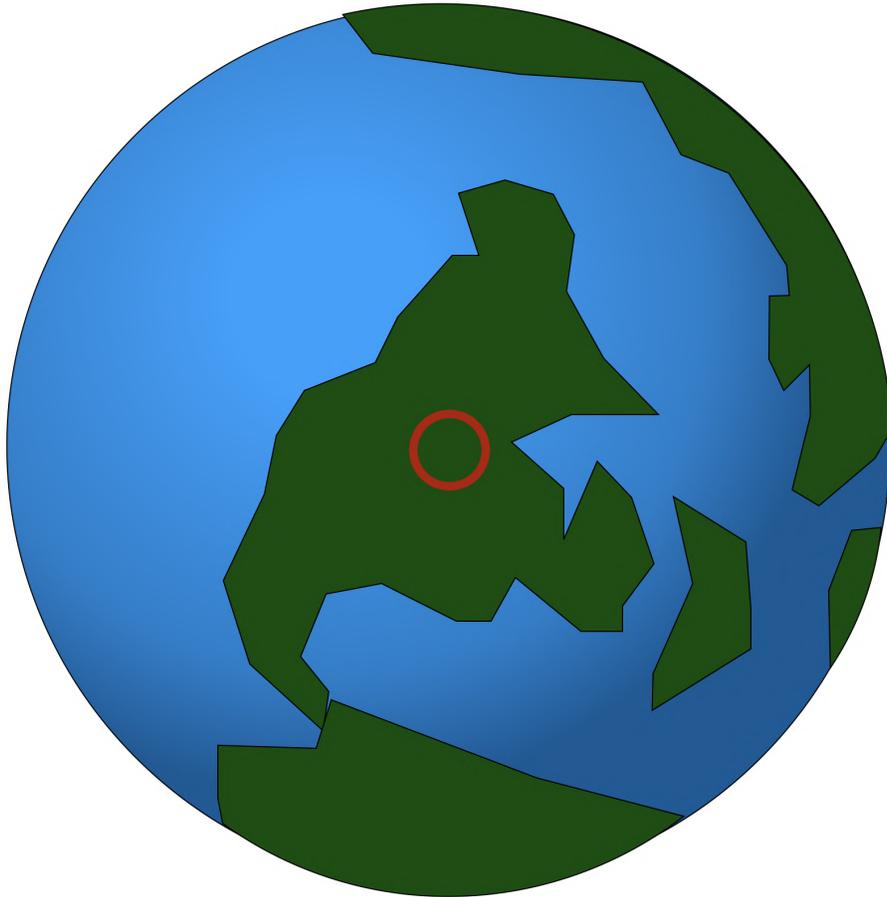

The correct approach instead would be to use the interior Schwarzschild metric.

Ok, but let's change reality a little bit and assume the same (identical) situation, with the only tiny difference that the radius of Earth is less than 8.87 mm:

$$R < 8.87 \text{ mm}$$

In order to make it true, we would have to (somehow) compress the

entire mass of our planet inside a volume equivalent to a ball with radius $R < 8.87mm$.
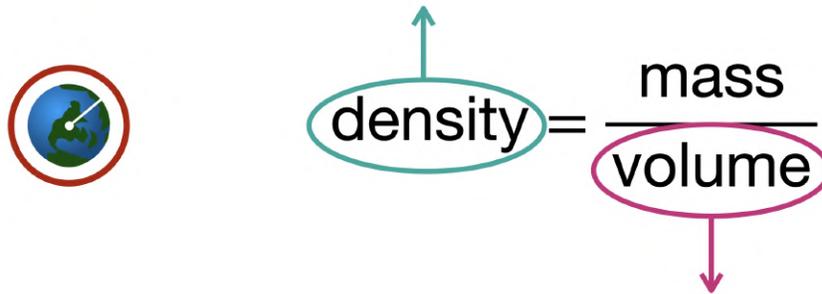
This would be an extremely dense (mass/volume $>> 1$) object, but if it were created, then all of a sudden the metric $(*)$ turns out to be a (almost) consistent physical model, since we're located outside the body now. We will see the problem with this metric in a moment.

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 \\ 0 & 0 & \dfrac{4G^2M^2}{c^4} & 0 \\ 0 & 0 & 0 & \dfrac{4G^2M^2}{c^4}\sin^2\theta \end{pmatrix}$$

where $0 = g_{00}$ (time) and $\infty = g_{11}$ (radial)

In $(*)$, as you can see, the time component is zero, which means that time appears frozen to distant observers $(g_{00} = 0)$. The radial component diverges $(g_{11} \to \infty)$. However, this is not a true physical singularity. It is a coordinate singularity, meaning it appears due to the choice

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of coordinates. The spacetime curvature remains finite here, and the metric can be extended smoothly through r_S using better coordinates.

The true singularity, where curvature becomes infinite, is at $r = 0$.
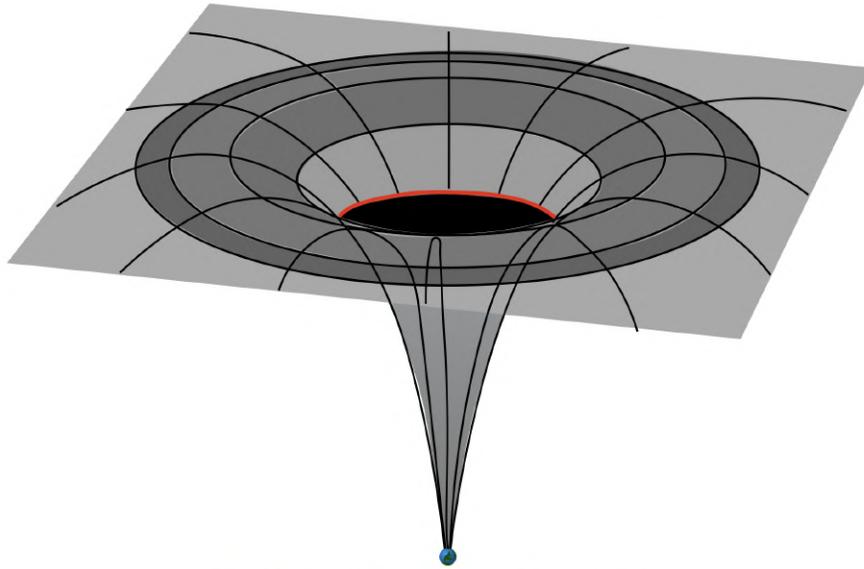

Caution: **do not** try this at home.

Congratulations! You just learned a scientific formula for producing a **black hole**!

So we can transform any massive object (including the Earth) into a black hole. All we gotta do is find a way of compressing it until it crosses the "physical limit" below which its radius is less than $\frac{2GM}{c^2}$.
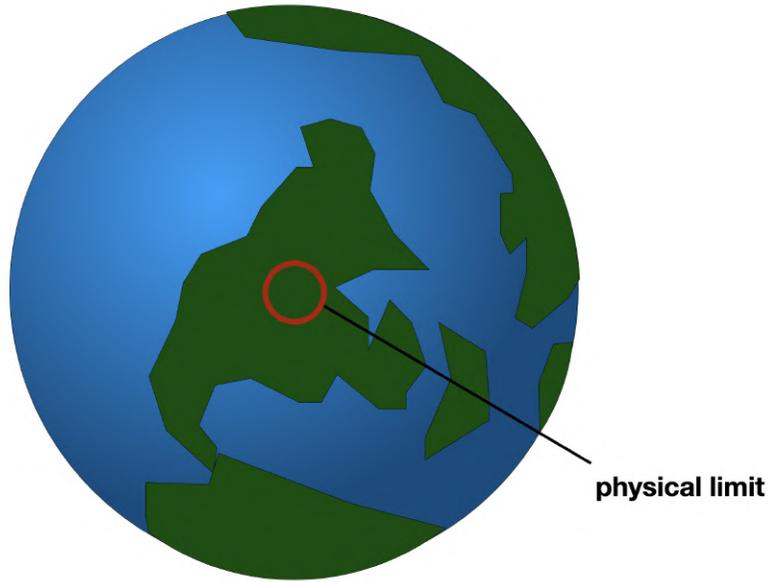
This "physical limit" is called the event horizon, and the limiting radius is the **Schwarzschild radius**:

$$R < \frac{2GM}{c^2} = r_S \text{ (Schwarzschild radius)}$$

physical limit = event horizon
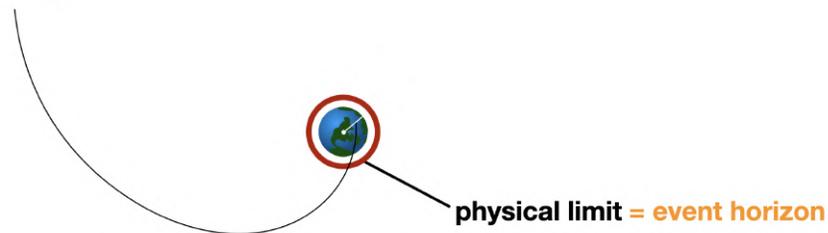
A question that naturally popped up in my mind when I just learned that was: *"So if I can turn any object with mass into a black hole, then what really counts in here is the ratio $\frac{mass}{volume}$. As long as the volume is tiny, I don't need a lot of mass in order to produce a singularity. But then, what happens when I want to transform a very little mass into a black hole by compressing it inside of a volume equivalent to a ball of radius less than the **Planck scale**?"*
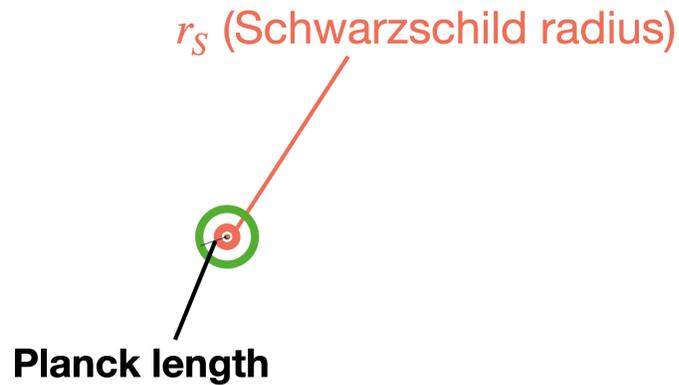
Planck scale
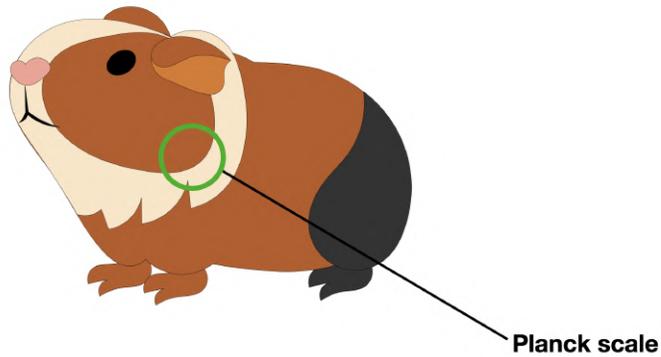
If the object's radius becomes smaller than its Schwarzschild radius $r_S = \frac{2GM}{c^2}$, it forms a black hole, regardless of how large or small the mass is.

Planck scale
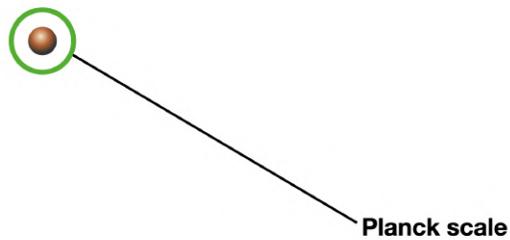
For small masses, the Schwarzschild radius becomes extremely tiny. If r_S drops below the Planck length $\left(l_P \sim 1.6 \cdot 10^{-35} m\right)$, then the black hole description enters a regime where General Relativity and Quantum Mechanics blend together, and **quantum gravity** effects dominate.

r_S (Schwarzschild radius)
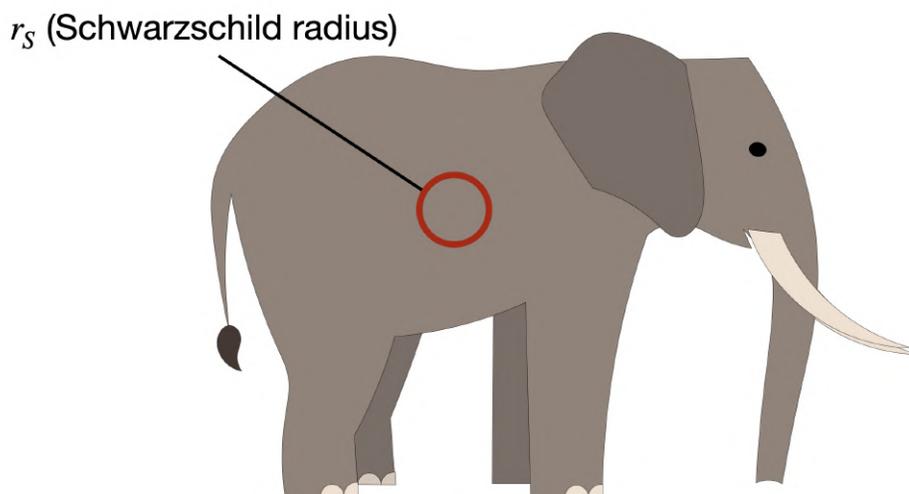
Currently, nobody really knows what happens in this *"quantum gravity realm"*. The only things we can say for sure is that classical General Relativity is not valid anymore, and that Quantum Mechanics and gravity must both (somehow) be taken into account.

r_S (Schwarzschild radius)

Planck length

Such weird objects are called *Planck-scale black holes* or *micro black holes*, and that's where speculations about them come from.

r_S (Schwarzschild radius)

Planck scale
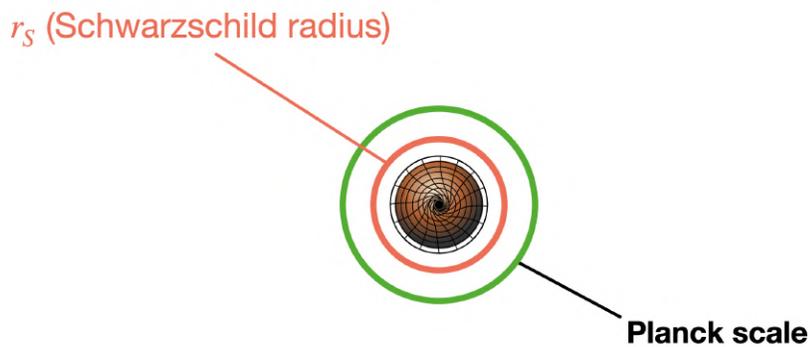
Let's see a concrete example.

Concrete Example

Imagine we have an *electron* $\left(M_{e^-} \approx 9.11 \cdot 10^{-31}\,kg\right)$. If we want to turn it into a black hole, we must compress it to a tiny volume equivalent to a little sphere with radius less than its Schwarzschild radius.

Let's calculate its Schwarzschild radius, then:

$$r_S = \frac{2GM_{e^-}}{c^2} \cong \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 9.11 \cdot 10^{-31}}{(2.99 \cdot 10^8)^2} \cong 1.35 \cdot 10^{-37} \text{ m}$$

This is way smaller than the Planck length:

$$1.35 \cdot 10^{-37} \text{ m} \cong r_S < l_P \cong 1.6 \cdot 10^{-35} \text{ m}$$

Therefore, the idea of forming a black hole out of an electron is *meaningless* without a proper theory of quantum gravity.

Conclusion

The last thing I'll say about it is that for our planet Earth (or any other ordinary massive object that's not a black hole), the Schwarzschild radius $r_S = \frac{2GM}{c^2}$ is much smaller than its physical radius R. So the event horizon is located inside the body and has no physical effect, for all non-black hole objects ($r_S < R$). Only when we imagine compressing the object so that its radius R becomes smaller than or equal to its Schwarzschild radius r_S, does the horizon find itself outside the matter distribution, and the object becomes a black hole.

<u>non-black hole</u>

mass

"event horizon"

$(r_S < R)$

mass

"event horizon"

$(r_S > R)$
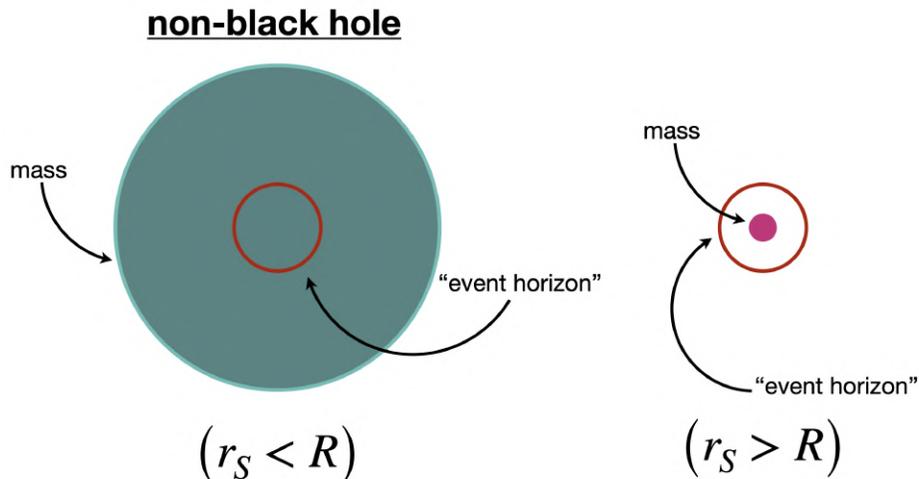
Today, we covered two of the most important solutions of Einstein's field equations: the Minkowski and Schwarzschild metrics. Below you will find a list of famous solutions of Einstein's equations. And if you want a deeper description of each, check out the following amazing material called "Catalogue of Spacetimes".

Some of the Famous Solutions

1. Minkowski

2. Schwarzschild

3. Reissner–Nordström

4. Kerr

5. Kerr–Newman

6. de Sitter (dS)

7. Anti-de Sitter (AdS)

8. FLRW (Friedmann–Lemaître–Robertson–Walker)

9. Kasner

10. Taub–NUT

11. Gödel

12. Plane gravitational waves

13. Lemaître–Tolman–Bondi (LTB)

14. Einstein static universe

15. Vaidya

16. Bianchi cosmologies (types I–IX)

17. Morris–Thorne wormhole

18. Alcubierre warp drive

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typos or things to improve, let us know as well. Your feedback
is very important to us. We're working hard to deliver the best
material possible. Contact us at: dibeos.contact@gmail.com