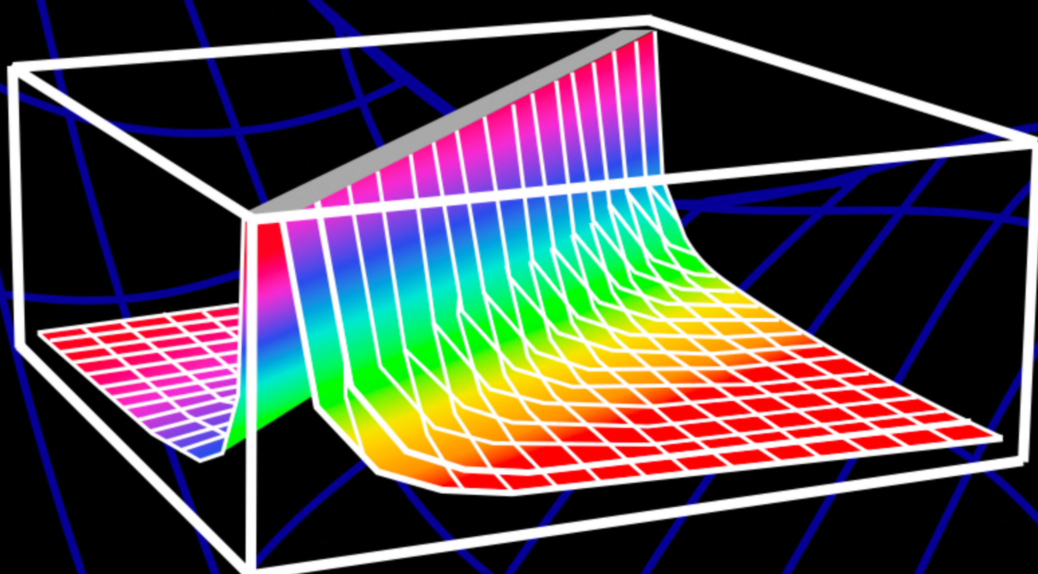
The Roadmap to

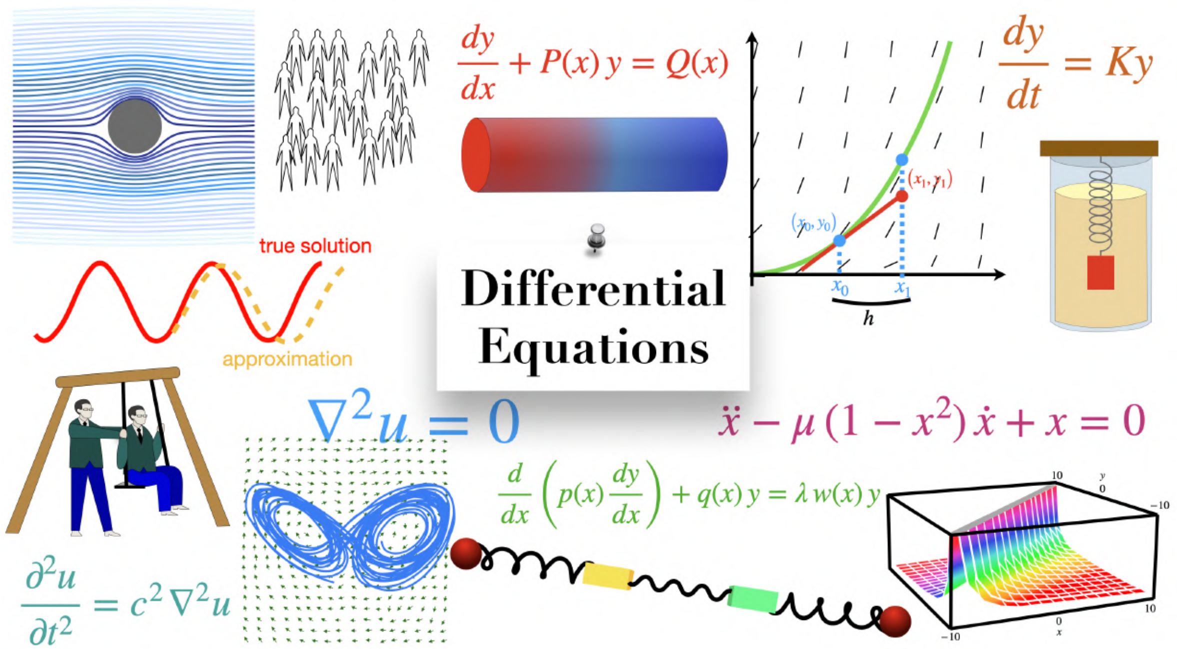
Differential Equations

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$



The Roadmap to Differential Equations

by DIBEOS



"Thus the partial differential equation entered theoretical physics as a handmaid, but has gradually become mistress" – **Albert Einstein**

Do not forget to check out our catalogue of **PDFs right here** You might find something that interests you!

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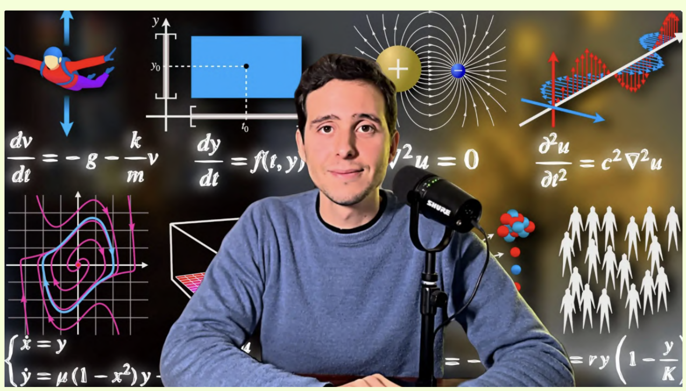
This PDF is a deeper look at the material discussed in the following YouTube video:



What it Takes to Master Differential Equations.

We highly recommend watching the video first to get a basic understanding, and then reading this PDF.

Introduction

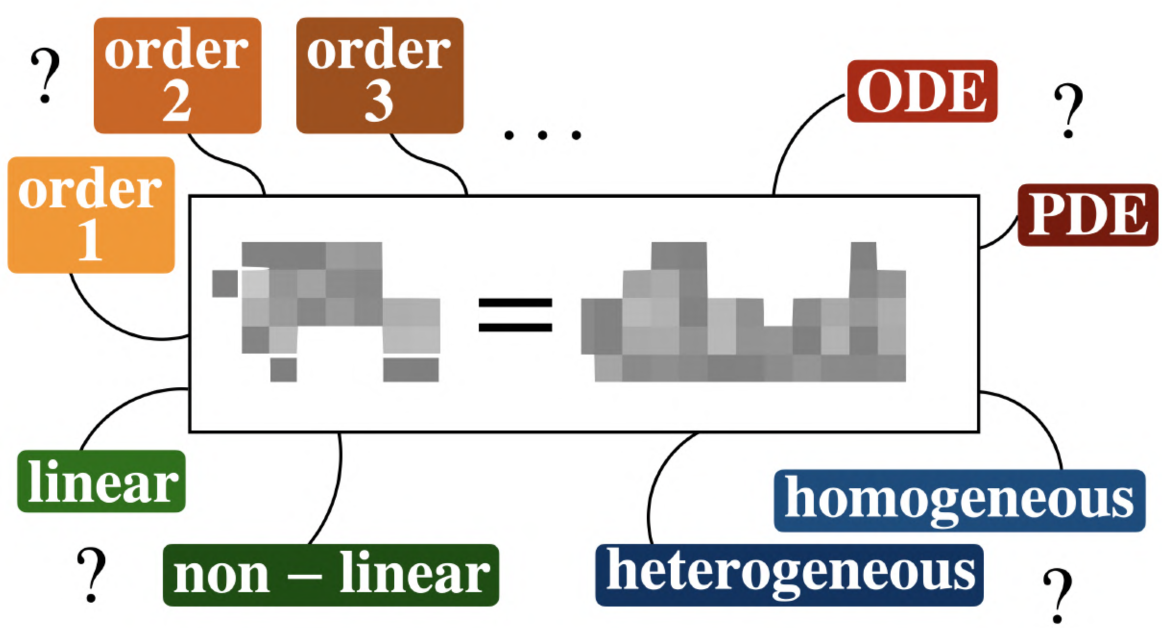
What does it really take to master differential equations?

Here we will see 6 steps that will serve as a roadmap to help you master differential equations (DEs) on your own pace, and with as much depth as you like.

(1) The Language of DEs

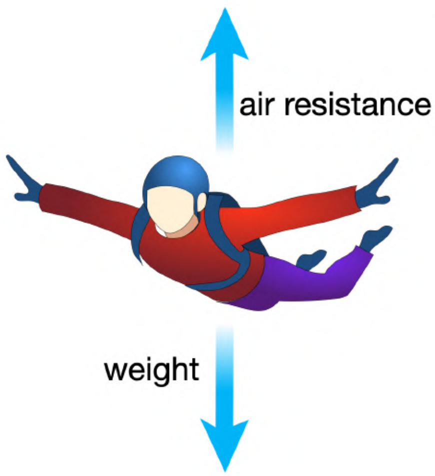
The first thing you need to master is the language of differential equations.

When you look at a DE, you should be able to immediately identify whether it's an **ordinary** or **partial differential** equation (**ODE** or **PDE**), whether it's **linear** or **nonlinear**, whether it's **homogeneous** or **heterogeneous**, and what its **order** is: first, second, third, . . .



Why does it matter? Because your ability to choose the right method depends on how you classify the equation. But that's not enough. You also need to know what kind of problem you're trying to solve. Is it an **initial value problem** (**IVP**)?

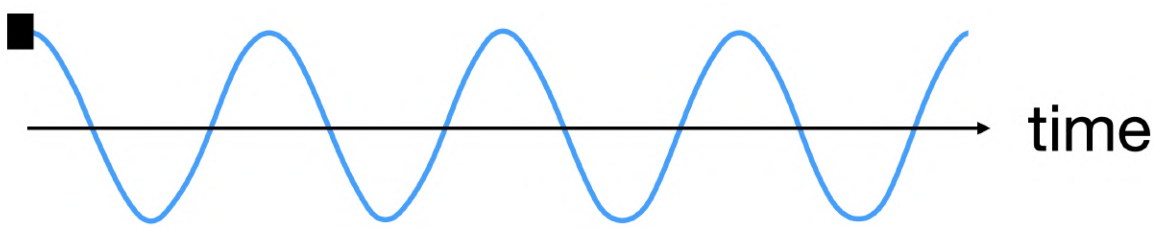
initial value problem (IVP)?



$$\boxed{\frac{dv}{dt} = -g - \frac{k}{m}v}$$

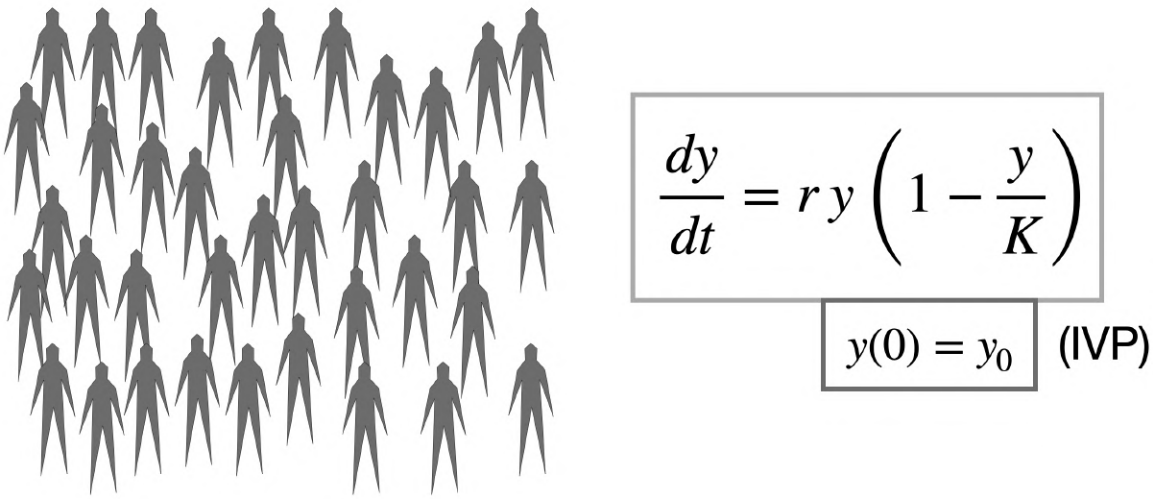
$$\boxed{v(0) = 0 \quad \text{(IVP)}}$$

Like a free fall with a fixed initial velocity?



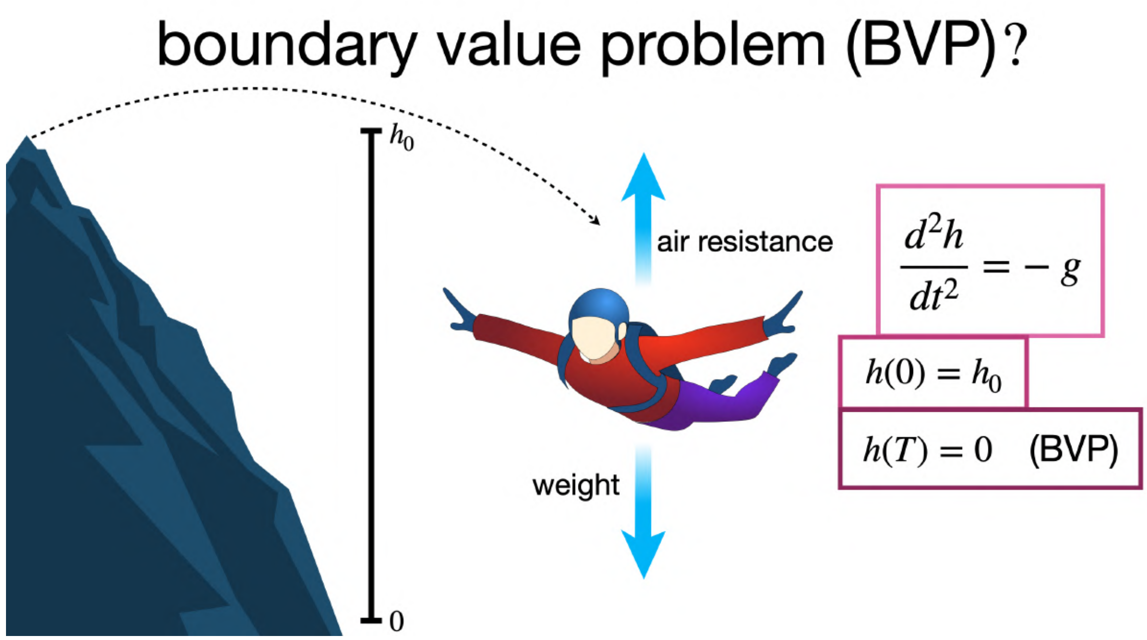
$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}} \quad \boxed{\begin{array}{c} u(x,0) = f(x) \\ \hline \dfrac{\partial u}{\partial t}(x,0) = g(x) \end{array}} \quad \text{(IVP)}$$

Or a wave oscillating with initial and final time described only by functions of space?

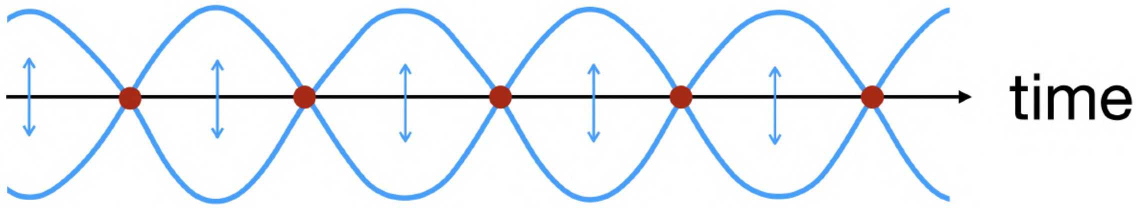


Or maybe it is a population growth model with a fixed initial number of members?

Is it a **boundary value problem** (**BVP**)?

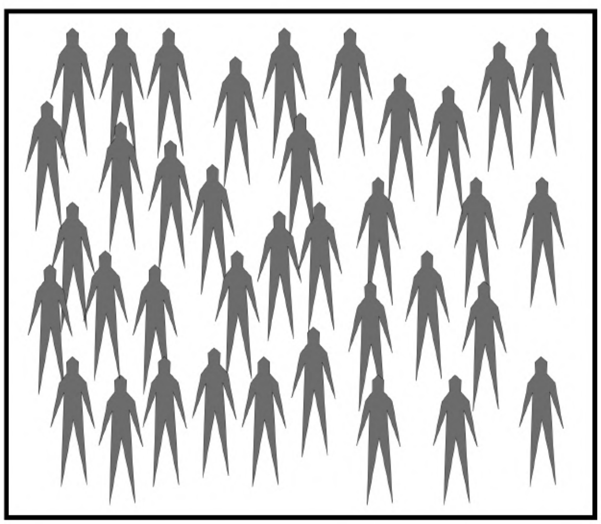


Like a free fall with fixed initial altitude and final destination?



$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}} \quad \boxed{u(0,t) = 0} \quad \boxed{u(L,t) = 0} \quad \text{(BVP)}$$

Or a stationary wave with fixed boundaries?

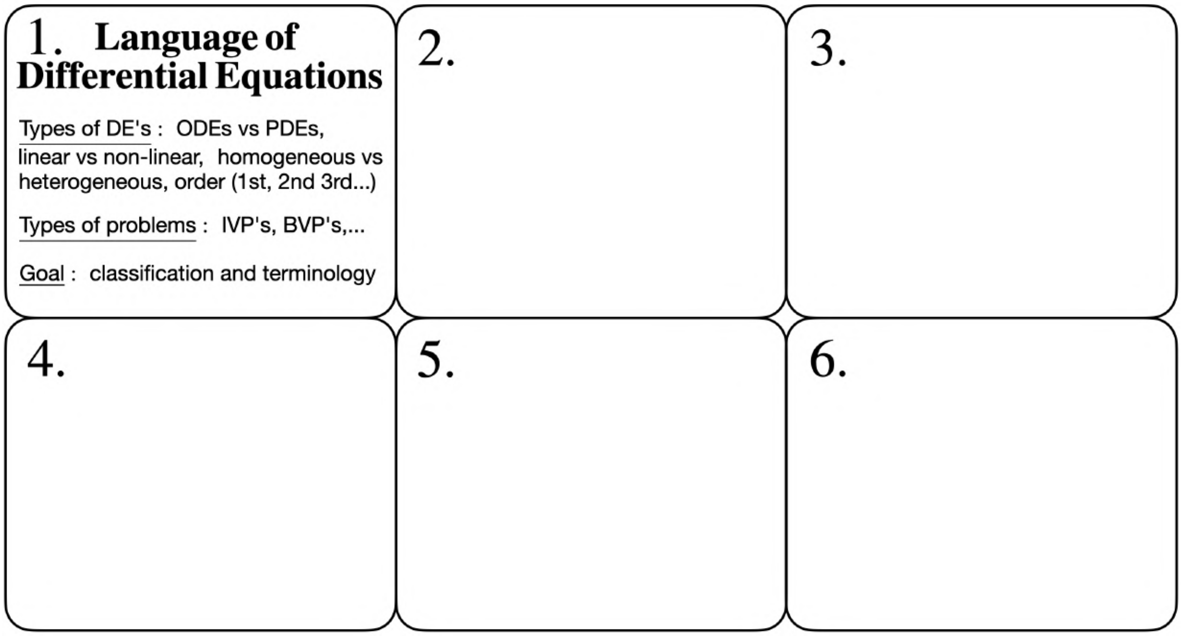


$$\boxed{\frac{d^2 y}{dx^2} + r\,y\left(1 - \frac{y}{K}\right) = 0}$$

$$\boxed{y(0) = y(L) = 0} \quad \text{(BVP)}$$

Or a population growth that has a physical limitation in space? Like walls surrounding it.

Or maybe it's possibly both? Initial AND boundary value problem. Or maybe neither! Understanding first the type of equation, and the type of problem, gives you the first part of the roadmap to look for solutions.



So that's your goal in the first stage: Get fluent in DE's *classification* & *terminology*.

(2) Exact Solutions (When Possible)

CLICK HERE TO CONTINUE READING