

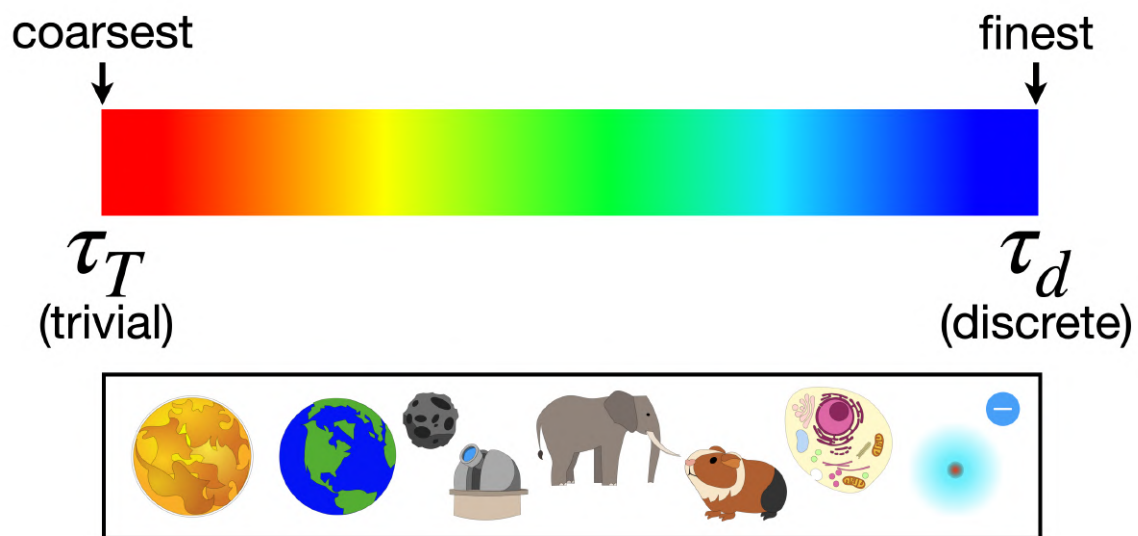
The Core of **Topology**





What Makes a Space TOPOLOGICAL?

by DIBEOS



“Topology is precisely the mathematical discipline that allows the passage from the local to the global.” – René Thom

Do not forget to check out our catalogue of [PDFs right here](#) You might find something that interests you!

Contents

Introduction	3
Open Sets	5
Why $\emptyset$ & X?	12
Building a True Topology	14
What is the Correct Topology on X?	17
Practice	19
Topological Fineness	20
Example: Finite Set	20
Example: Infinite Countable Set	27
Fun Fact	34
Infinite Uncountable Sets (finally)	36
Conclusion	47
Practice (Exercises)	49
Appendix	73

This PDF is a deeper look at the material discussed in the following YouTube video:

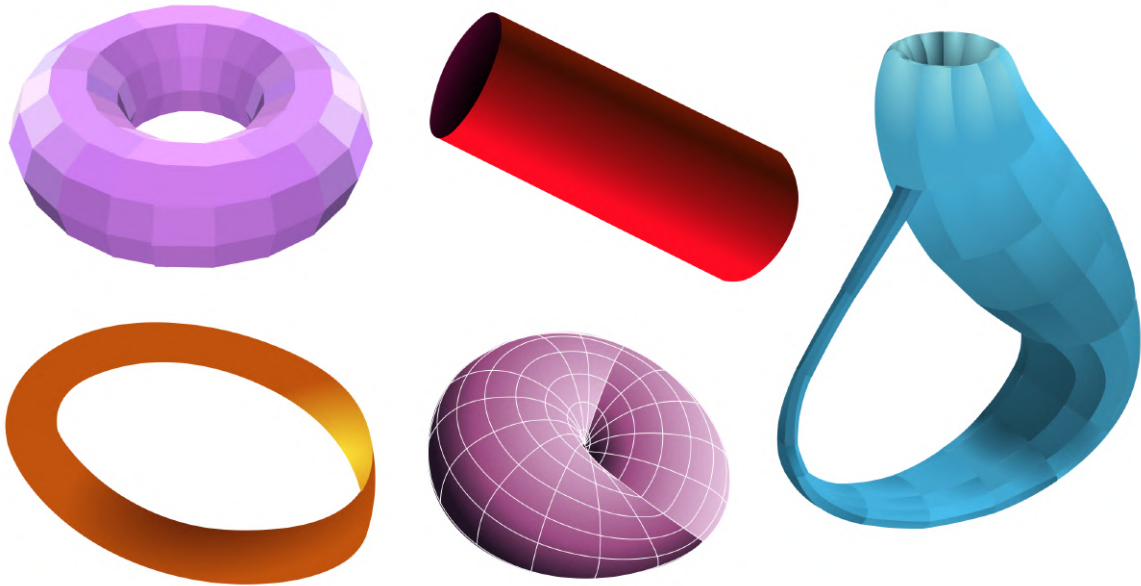


The Simple Problem Mathematicians Got Wrong.

We highly recommend watching the video first to get a basic understanding, and then reading this PDF.

Introduction

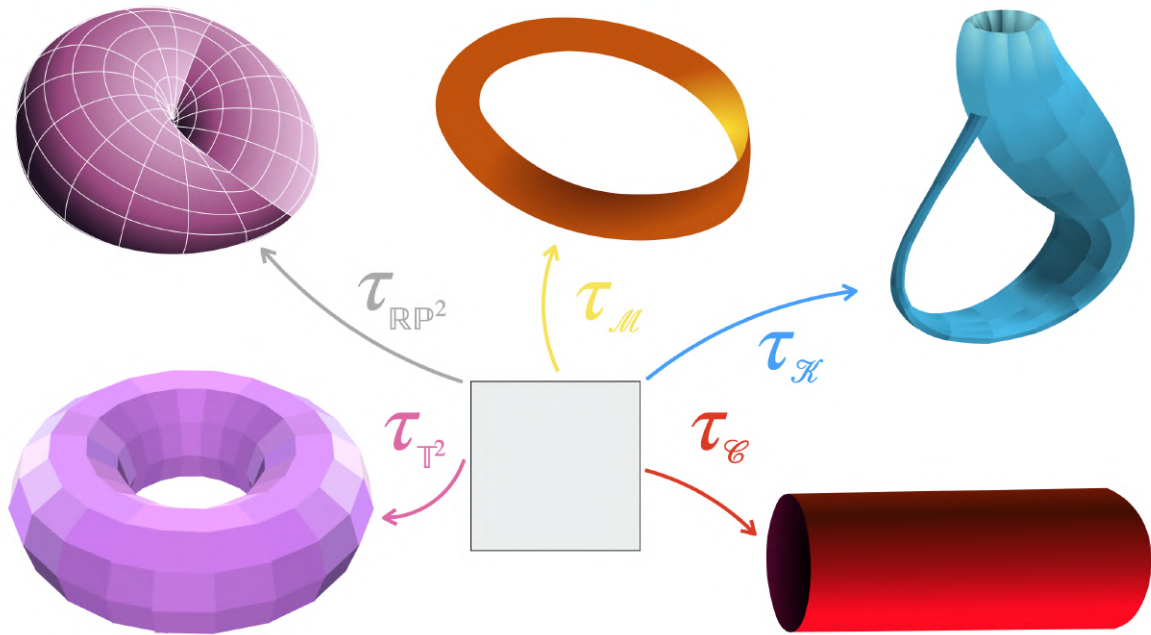
When you think of topology as a whole, usually shapes like these come to mind:



But how can we create them?

All you gotta do is choose the correct topology for each one of them. But what is *a topology* after all? It's a set of *open sets*. Ok... but what does the word *open* mean in this context? Its meaning is much more abstract than you might think.

Did you know that all of these shapes can be created from a simple square?



When I was learning these concepts, I couldn't really understand the connection between all these *elastic shapes* and *open sets*. And to be honest, making this [PDF](#) and the [YouTube video](#), forced me to research these concepts all over the place. And this really helped me clarify a lot of things! I hope we will be able to do the same for you.

So, let's start with a **bag of socks**.



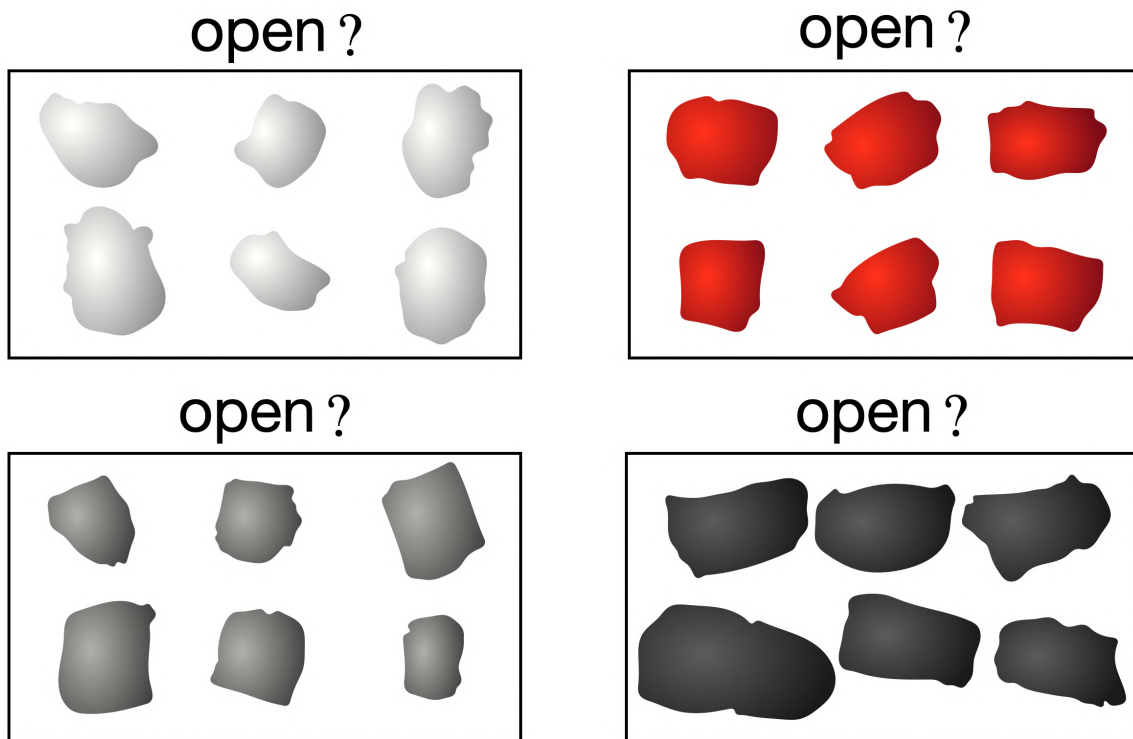
Open Sets



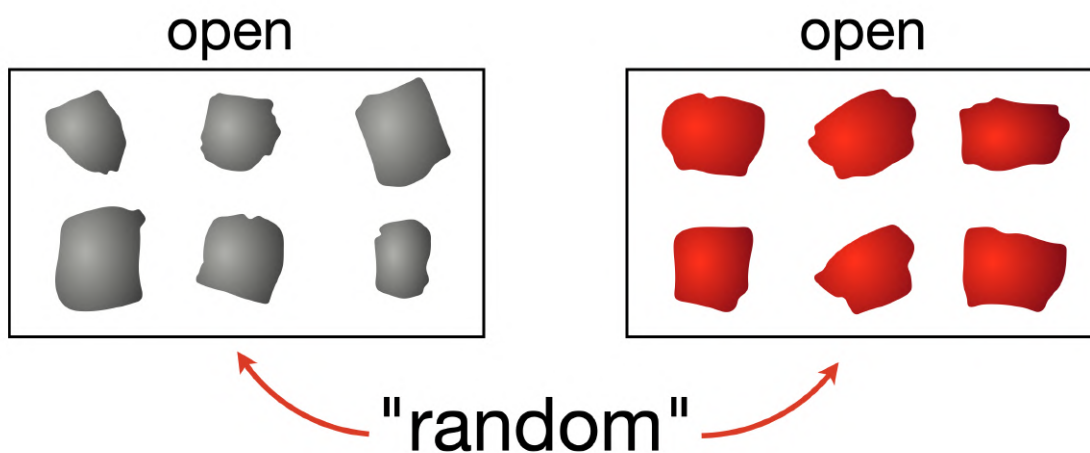
This bag of socks is our set X . We have subsets of *black*, *grey*, *white* and *red* socks.



Are any of these *open*?

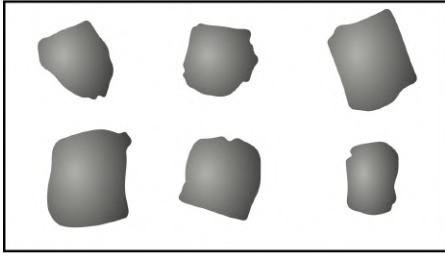


Well, let's just "randomly" decide that the following two subsets of my bag of socks are going to be called *open* (whatever that means...):



And this is the first step we can take to construct a topology on X . We can start with any collection of subsets of X that we want to call *open*, as long as 3 very important axioms are satisfied down the road.

open



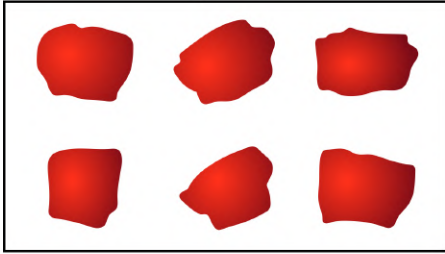
Axioms:

O_1 :

O_2 :

O_3 :

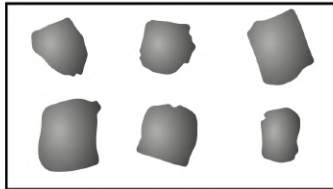
open



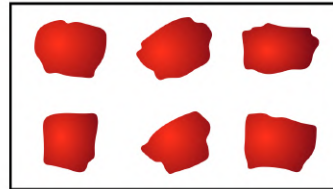
Let's apply the first one:

(I) If the grey and *red* socks are considered open subsets, then their union must also be considered open.

open



open



open

$$O_1 : \left[\begin{array}{|c|c|c|} \hline \text{grey sock} & \text{grey sock} & \text{grey sock} \\ \hline \text{grey sock} & \text{grey sock} & \text{grey sock} \\ \hline \end{array} \right] \cup \left[\begin{array}{|c|c|c|} \hline \text{red sock} & \text{red sock} & \text{red sock} \\ \hline \text{red sock} & \text{red sock} & \text{red sock} \\ \hline \end{array} \right] = \left[\begin{array}{|c|c|c|c|c|c|} \hline \text{red sock} & \text{red sock} & \text{red sock} & \text{grey sock} & & \\ \hline \text{red sock} & \text{red sock} & \text{red sock} & \text{grey sock} & & \\ \hline \text{grey sock} & \text{grey sock} & \text{grey sock} & \text{red sock} & \text{red sock} & \text{red sock} \\ \hline \end{array} \right]$$

(axiom)

But, when we unroll all the socks we find out a new feature about them. Some are long, and some are short.

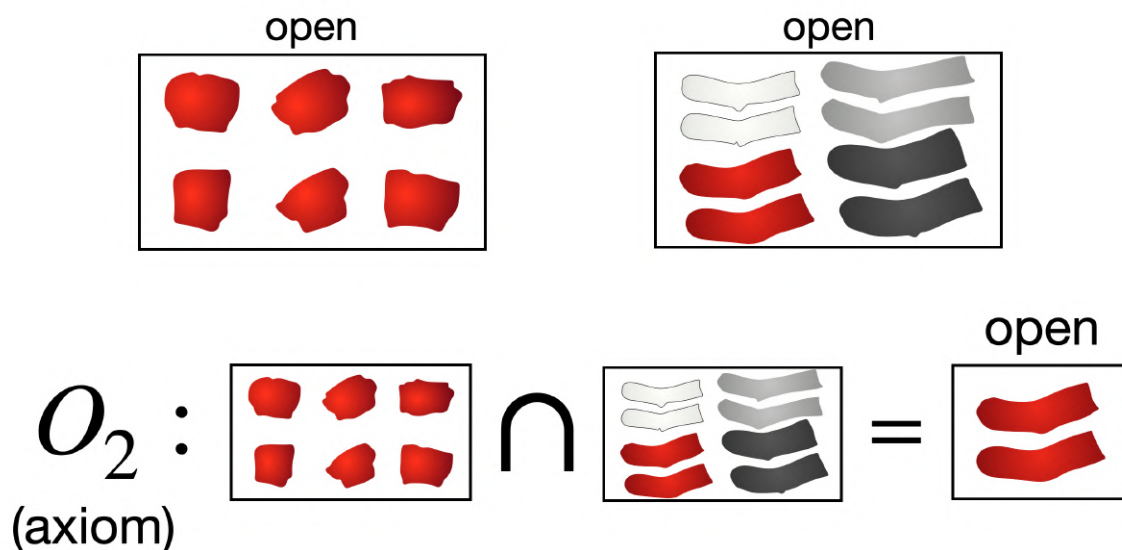


You know what?! Let's make another "random" decision of calling the subset of all long socks *open*, regardless of their color.

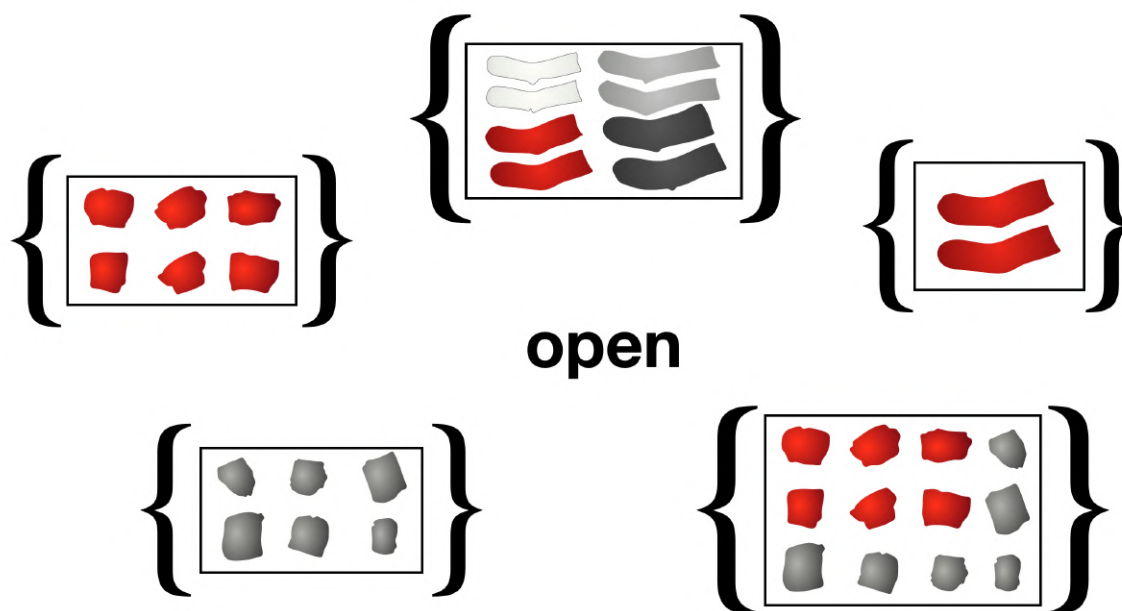


Then the second axiom would impose that, for example, the intersection

of the subset of red socks with the subset of long socks must also be considered open. (II)



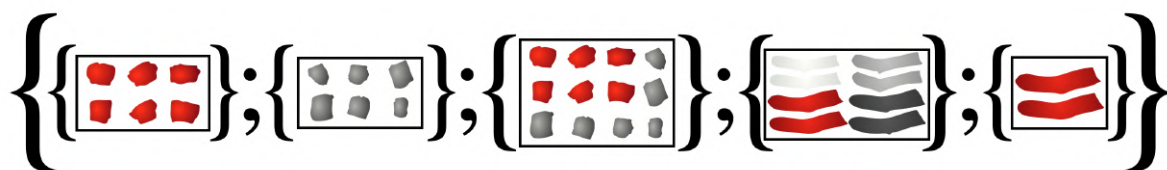
So far, we've decided that some groups of socks are going to be called *open*.



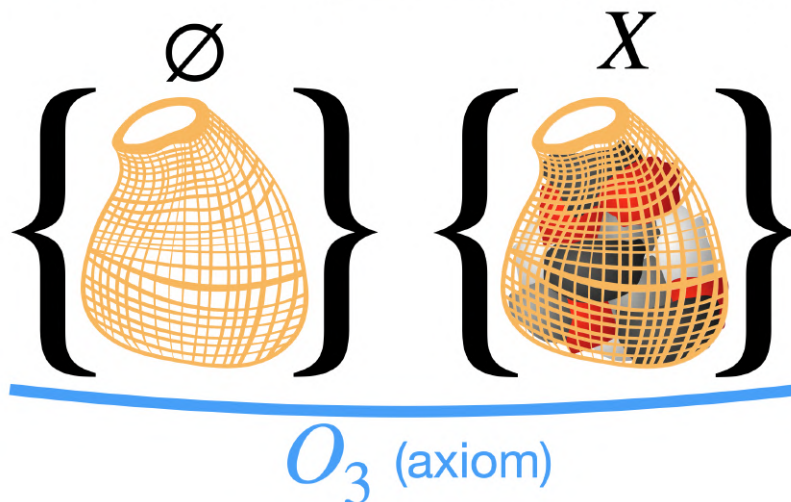
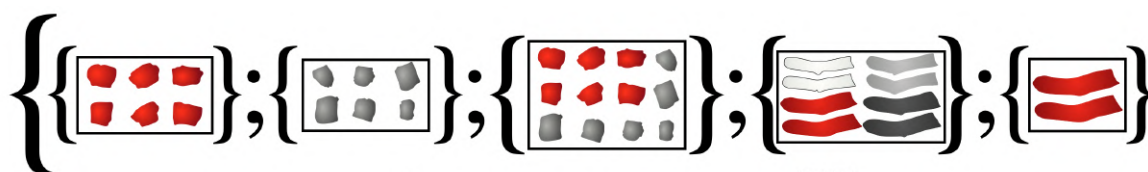
But don't try to assign any physical, or visual, meaning for the word open. All it means here is that these open sets, when put together, sat-

isfy axioms (I) and (II) that we've just seen.

All of these are open subsets in the bag of socks. Now, if I collect them into a set of sets,

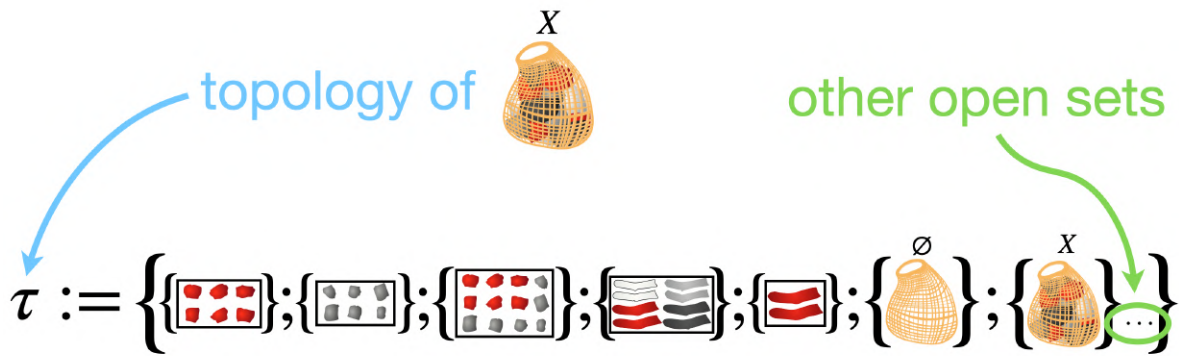


and then I also throw in the empty set $\emptyset$ and the entire bag of socks X (III),



then I've almost got something really special!

This set of sets (together with some other open subsets we will discuss in a moment) is usually denoted with the Greek letter τ (tau):



And this is called a **topology** of X .

Now, things are about to get interesting!!

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