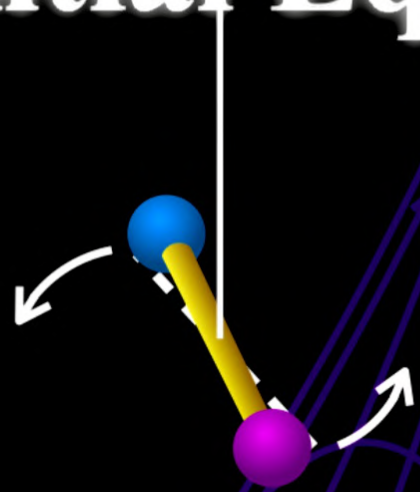
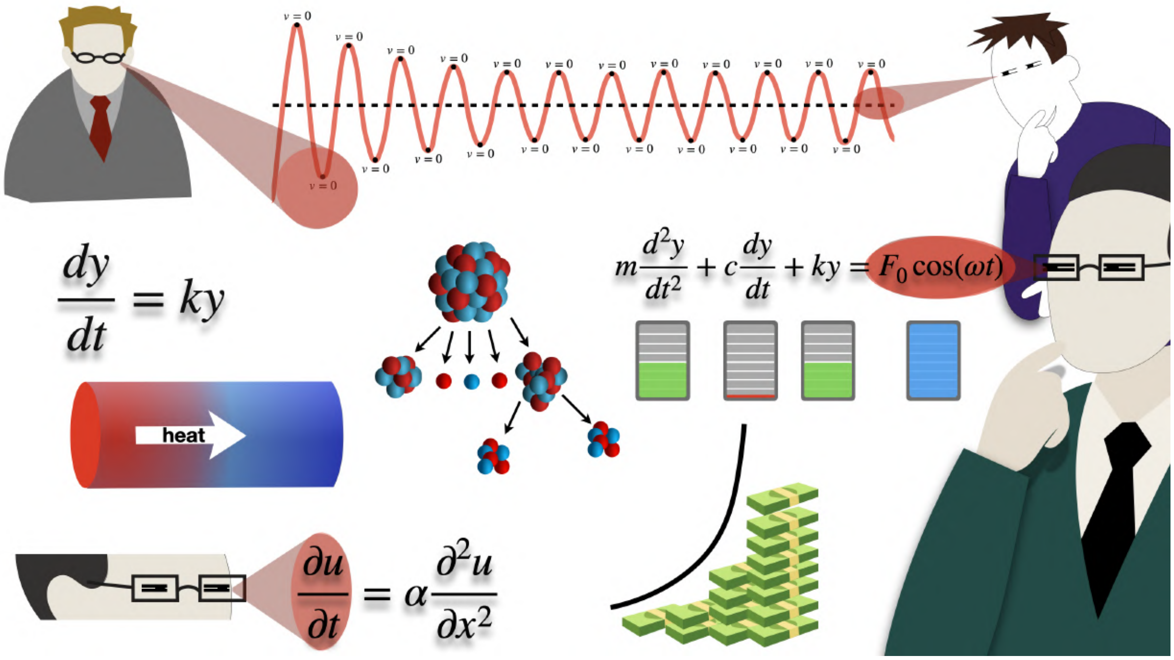
The Language of
Differential Equations



$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

The Language of Differential Equations

by DIBEOS



"The equations of mathematical physics express laws; they differ from ordinary empirical laws in this — they are differential equations." **– Henri Poincaré**

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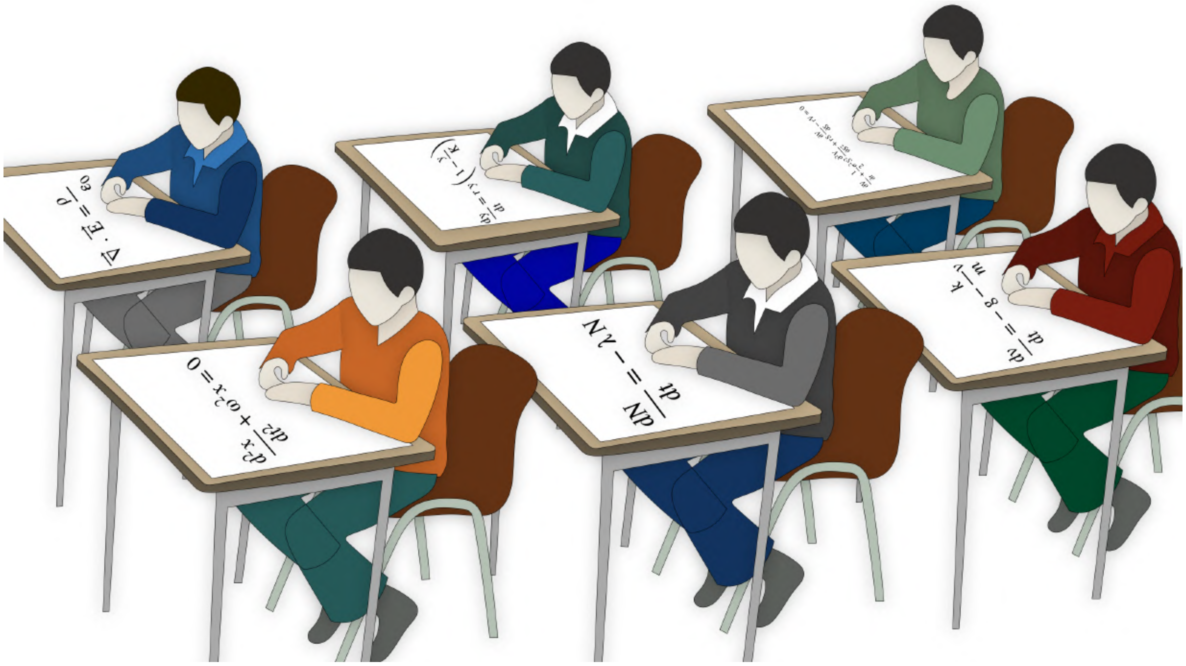
This PDF is a deeper look at the material discussed in the following YouTube video:

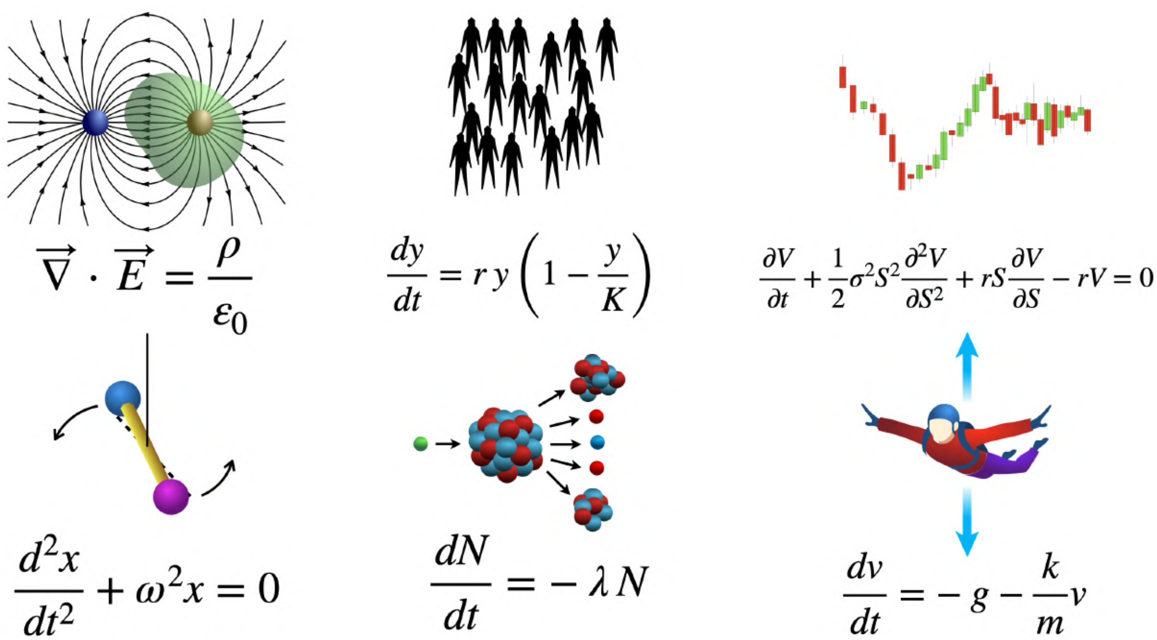
The Language of Differential Equations.

We highly recommend watching the video first to get a basic understanding, and then reading this PDF.

Introduction



Let's say you're given a differential equation. There is one thing you can say with *absolute certainty* (no matter what differential equation it is): something is changing!

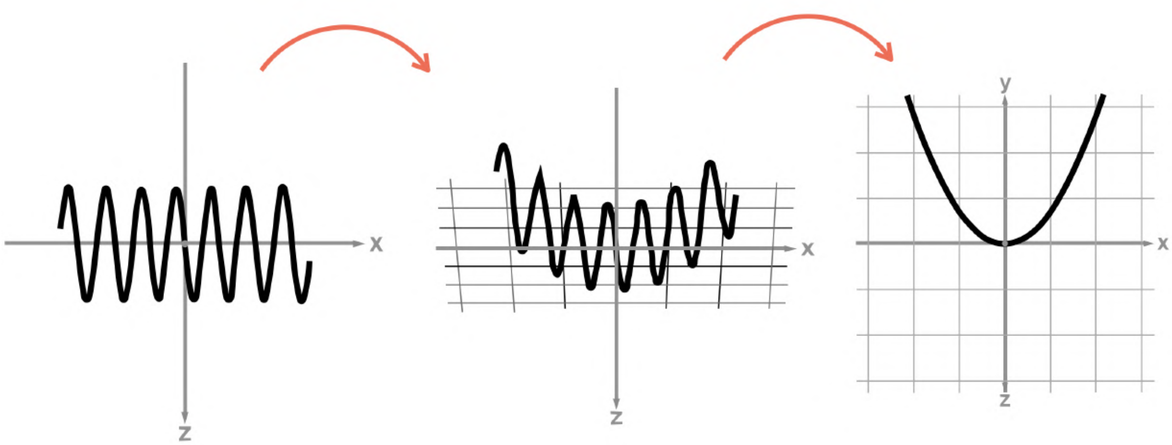


By definition, a differential equation involves **derivatives**, i.e. **rates of**

change.

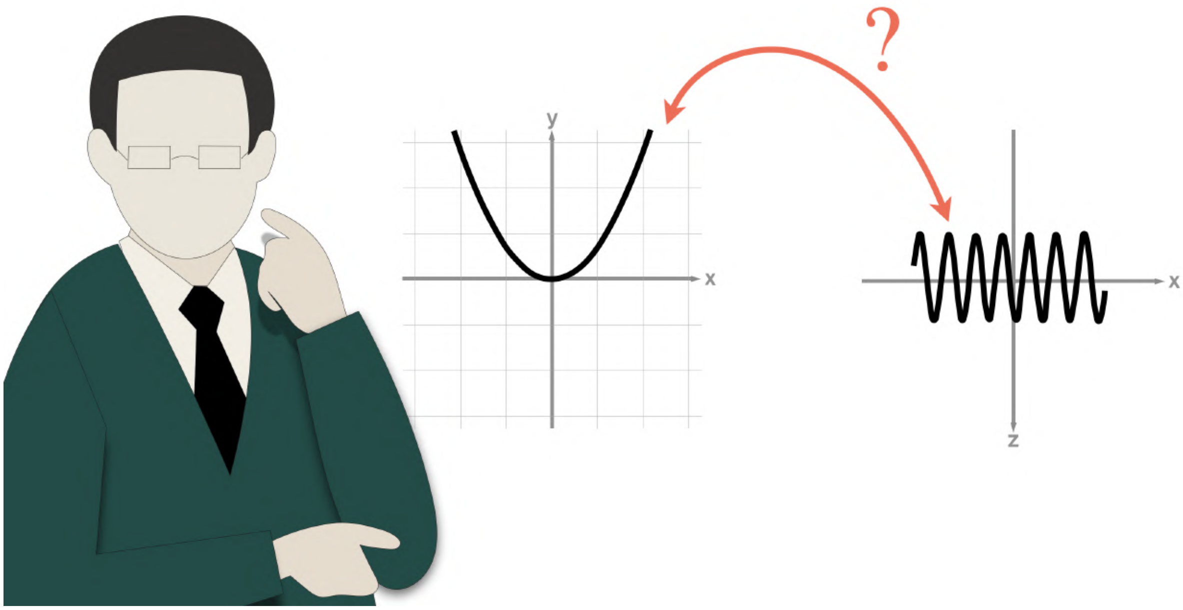
These changes can be with respect to *time*, to *space*, or simply with respect to *abstract variables* $x, y, z, \ldots$, if you're dealing with pure mathematics. The problem, though, is that change is relative.

From one perspective, one could say that the system changes in a periodic fashion, like a *sinusoid curve*. From another perspective, though, it's clearly *parabolic*, or *quadratic*.

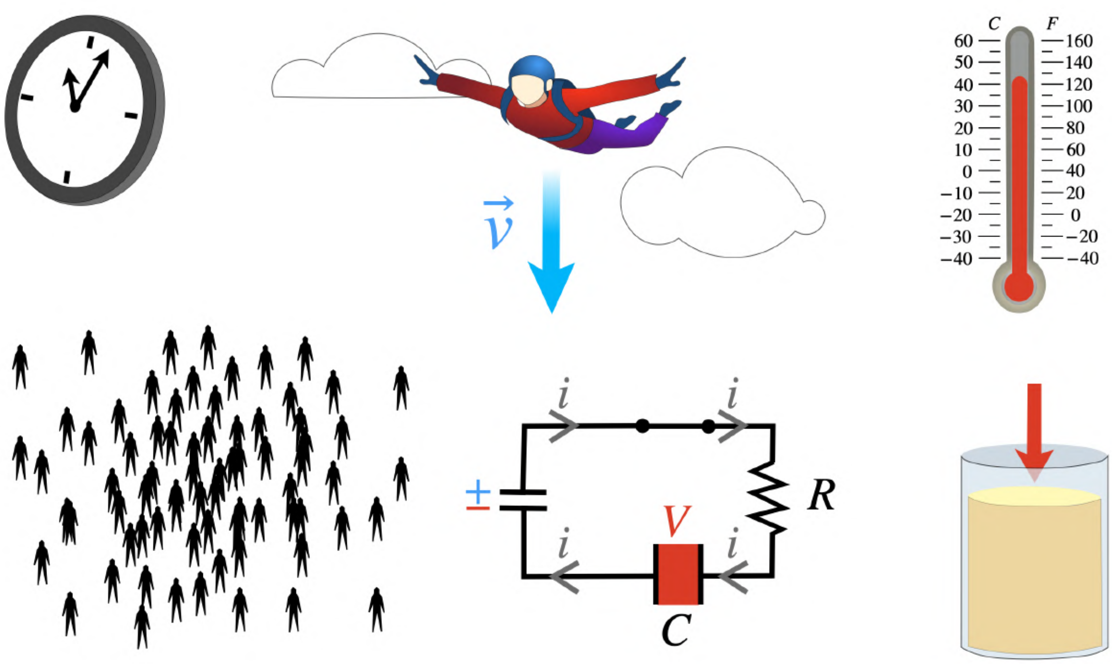


(click on the image above to "play" with the 3*D graph.)*

Which one is it, then? Well, again, it depends. The very first thing you need to learn, in order to "speak the language" of differential equations, is to identify what is changing with respect to what.



Is it a change in *time*? Or change in *space*? Or change in *population*? *Temperature*? *Velocity*? *Electric current*? *Pressure*? Or any other quantity?



Of course, if you're dealing with a problem in applied mathematics, things tend to be more tangible, and so it might be easier to spot what is the thing that is changing with respect to another thing. But even in pure math problems, you always have to start by answering these questions:

Which variable am I **tracking**?

With respect to what variable is it **changing**?

In order to find that out, you must train your eyes to look for these kinds of symbols and patterns (see below), because they all have one thing in common: they are describing some kind of *rate of change*:

$$\frac{dy}{dx} \quad \frac{dz}{dy} \quad \frac{d\psi}{d\theta} \quad y' \quad f'(x) \quad x' \quad \dot{y} \quad \dot{x} \quad \ddot{x} \quad D_x\left[f(x)\right]$$

$$D\left[g\right] \quad \frac{d^n y}{dx^n} \quad \frac{\partial u}{\partial x} \quad \partial_t\, y \quad \partial_{xy}\, w \quad \nabla u \quad \vec{\nabla} \cdot \vec{F} \quad \vec{\nabla} \times \vec{F}$$

$$\Delta y \quad \Delta x \quad \delta y \quad \nabla \quad \nabla^2 \quad \Box \quad \mathscr{L} \quad \frac{\delta S}{\delta \phi} \quad \delta \mathscr{L}$$

For more details on these symbols and their meaning, click right here.

Let’s see some of them in action. We will see 3 levels of difficulty here.

LEVEL 1

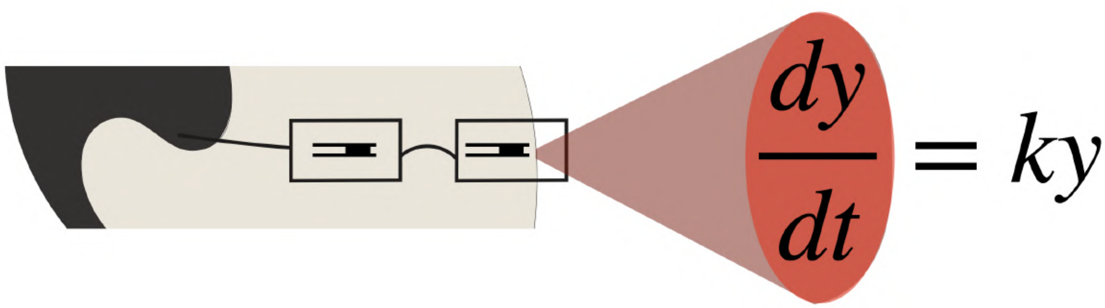
For the first one, we have the following differential equation:

$$\boxed{\frac{dy}{dt} = k\,y}$$

We start with a simple one. Let's say you were handed this equation, without any other details. How can you "read" it?

$$\frac{dy}{dt} = ky$$

Once you see this equation your eyes must immediately gravitate toward this region:



There is some quantity, called y, that is changing with respect to a parameter t, usually interpreted as time. So, imagine something changing over time.

CLICK HERE TO CONTINUE READING