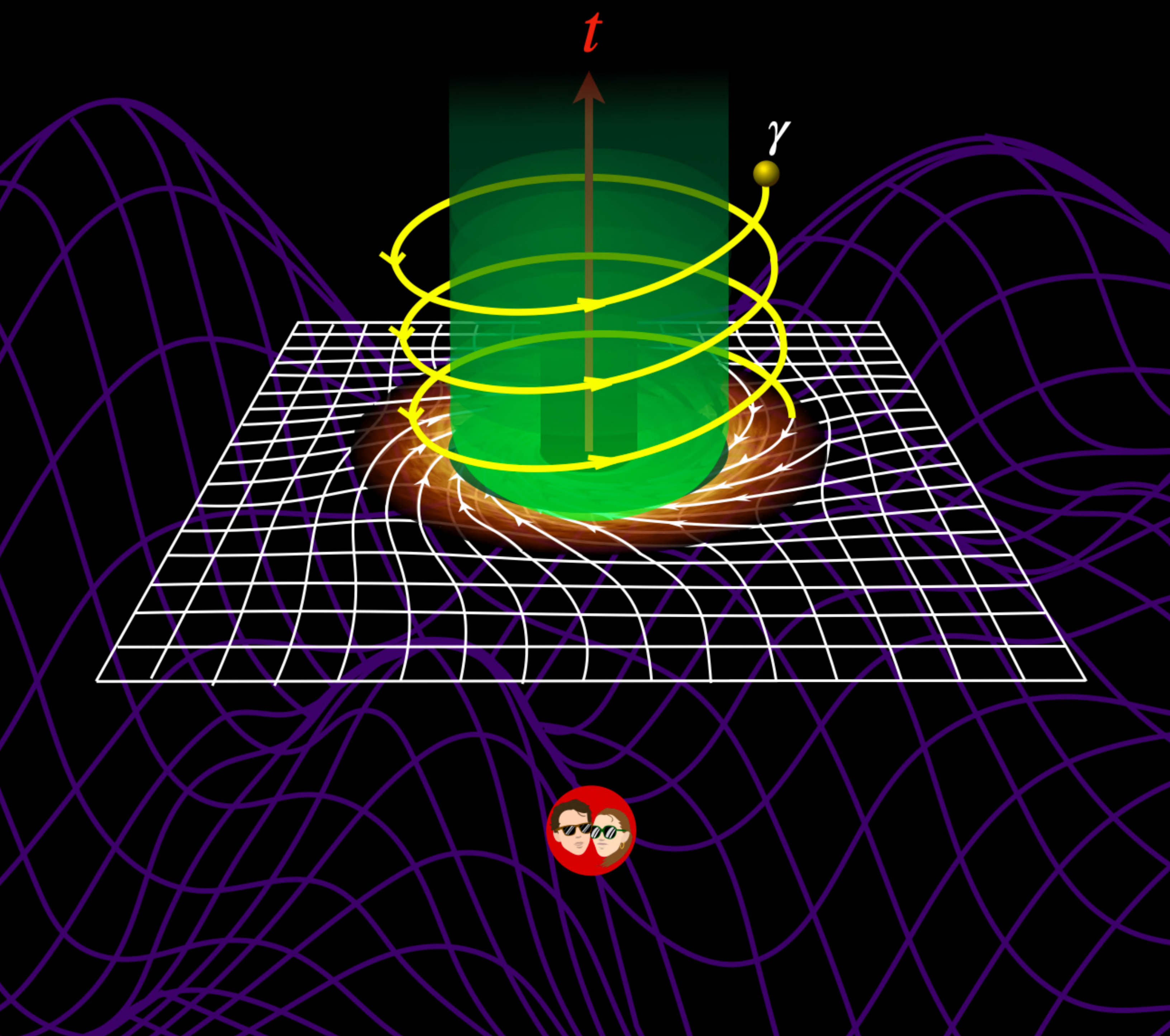
The Mathematics of a Rotating Black Hole



The Mathematics of a Rotating Black Hole

by DIBEOS

"The black holes of nature are the most perfect macroscopic objects there are in the universe. The only elements in their construction are our concepts of space and time." – **Subrahmanyan Chandrasekhar**

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Contents

This PDF is a deeper look at the material discussed in the following YouTube video:

The Mathematics of a Rotating Black Hole.

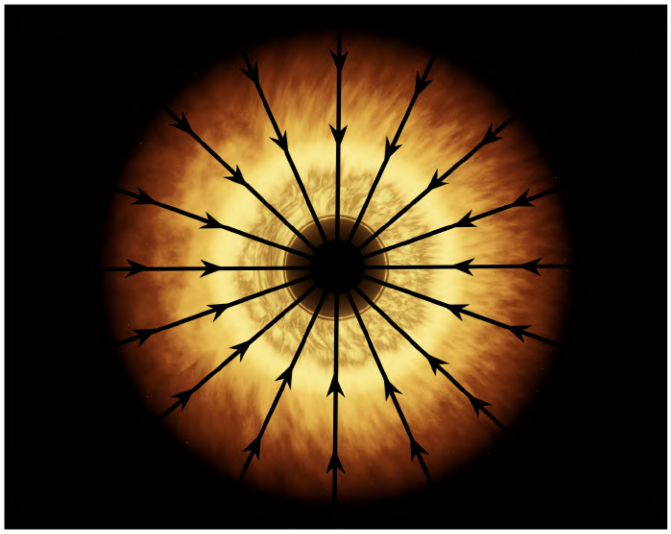
We highly recommend watching the video first to get a basic understanding, and then reading this PDF.

Introduction

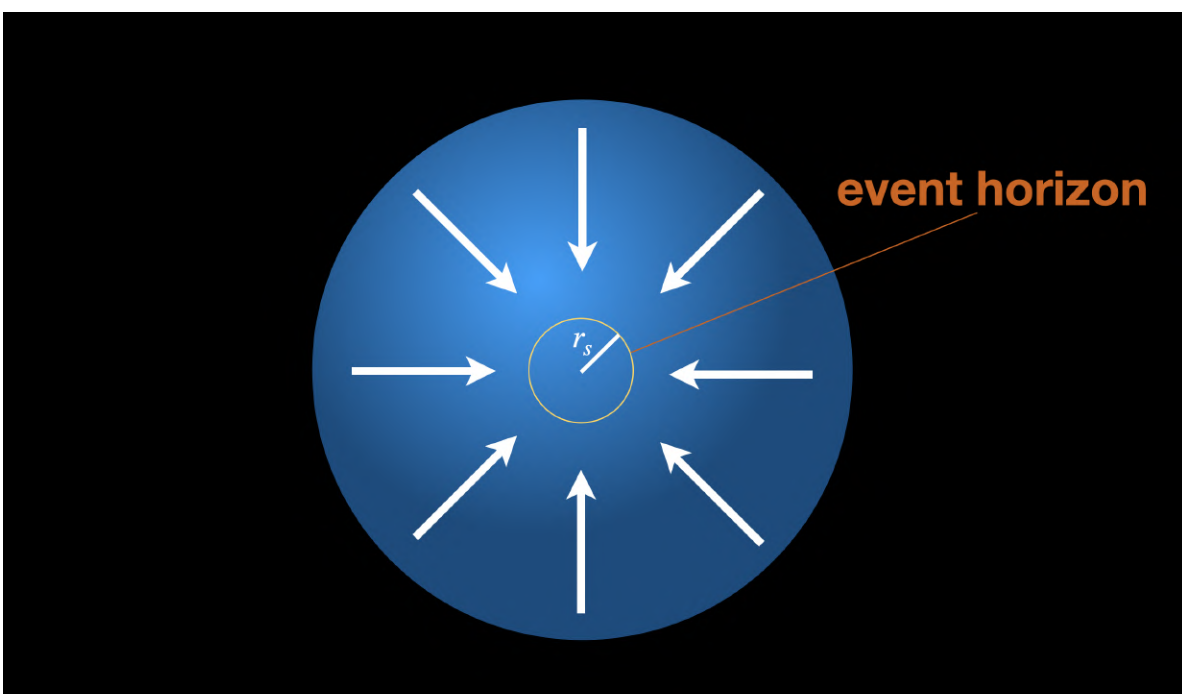
speed of light

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

Schwarzschild radius



It all started in 1915, when Karl Schwarzschild was able to find an exact solution to Einstein's field equations describing spacetime around a massive object.

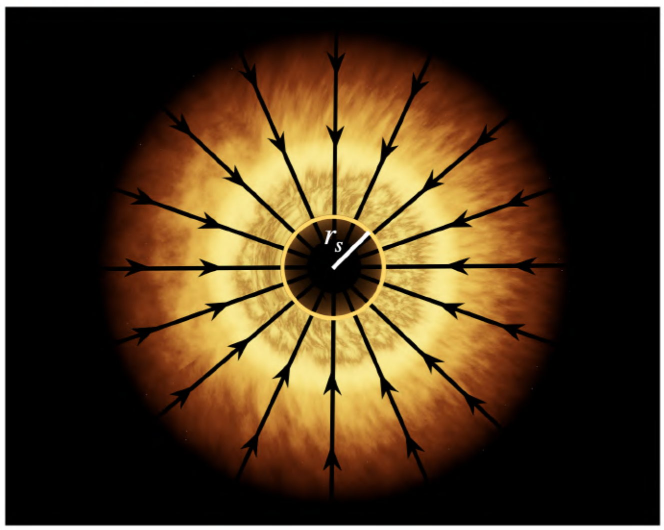


When the massive object is "compressed" within a radial distance from the origin that is less than or equal to a certain number that depends on the object's mass (now called Schwarzschild radius $r_S = \frac{2GM}{c^2}$) the body experiences gravitational collapse, and as a consequence a black hole is formed.

speed of light

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

Schwarzschild radius

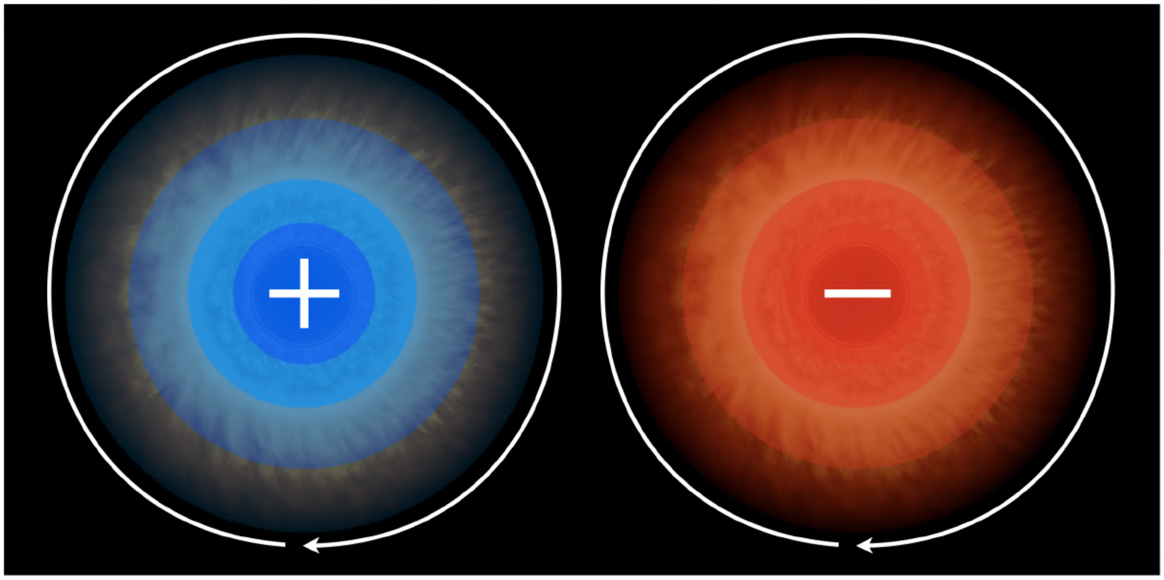


However, this solution does not take into consideration two very important characteristics of some black holes: *charge & rotation*.

speed of light

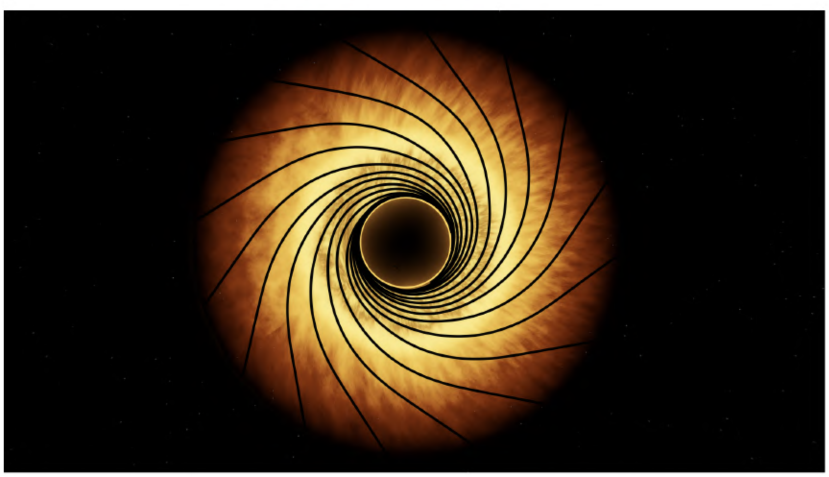
$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

Schwarzschild radius



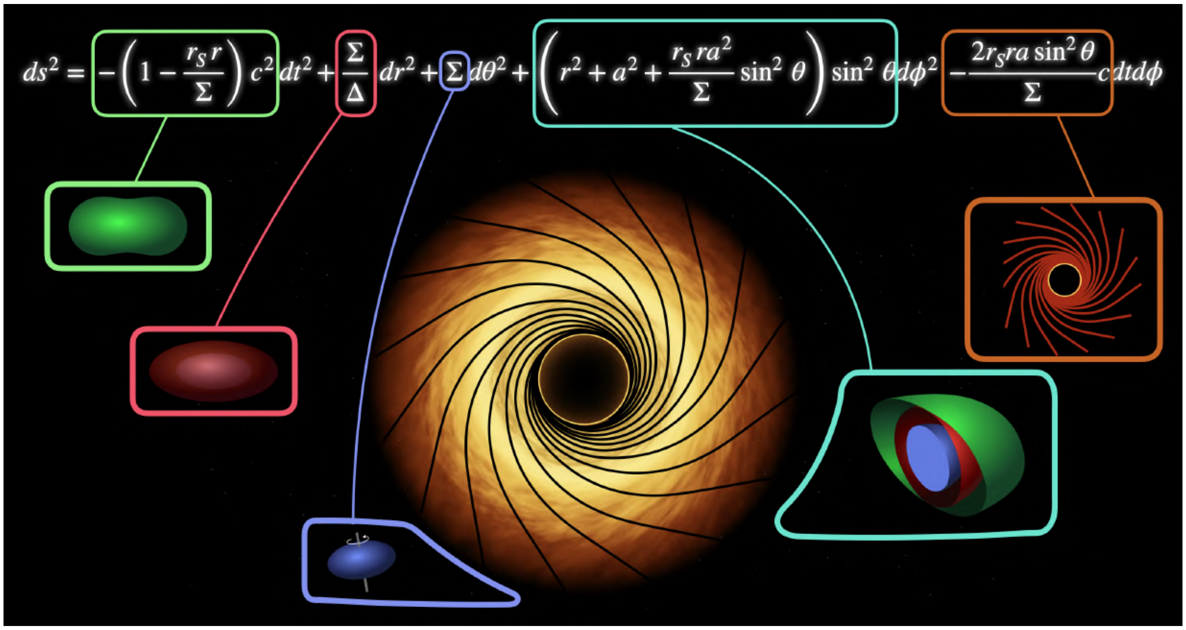
For a **rotating**, **uncharged black hole**, this is the solution to Einstein's field equations that describe spacetime around it:

Kerr Metric



$$ds^2 = -\left(1 - \frac{r_S r}{\Sigma}\right) c^2\, dt^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2 + \left(r^2 + a^2 + \frac{r_S r a^2}{\Sigma} \sin^2\theta\right) \sin^2\theta d\phi^2 - \frac{2 r_S r a \sin^2\theta}{\Sigma} c dt d\phi$$

This is called the **Kerr metric**, and today we will understand in detail each term in this mathematical expression so that when you look at this metric, what you actually see is this:



But first of all, *who is Kerr anyway? And why should you kerr?* (I mean, *care*)

Let us take a step back and map out our current knowledge on black holes.