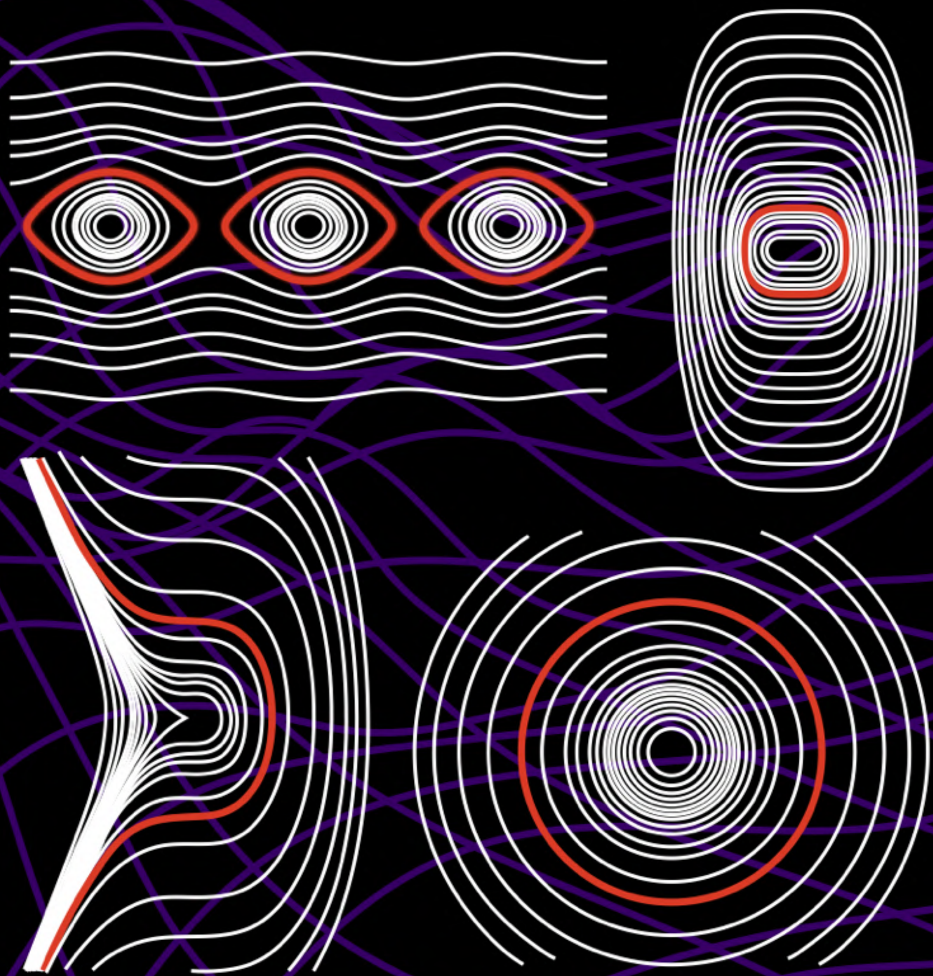
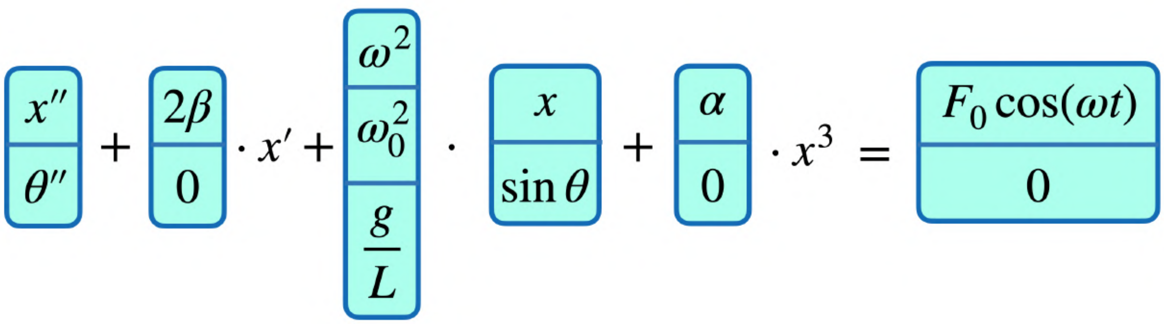
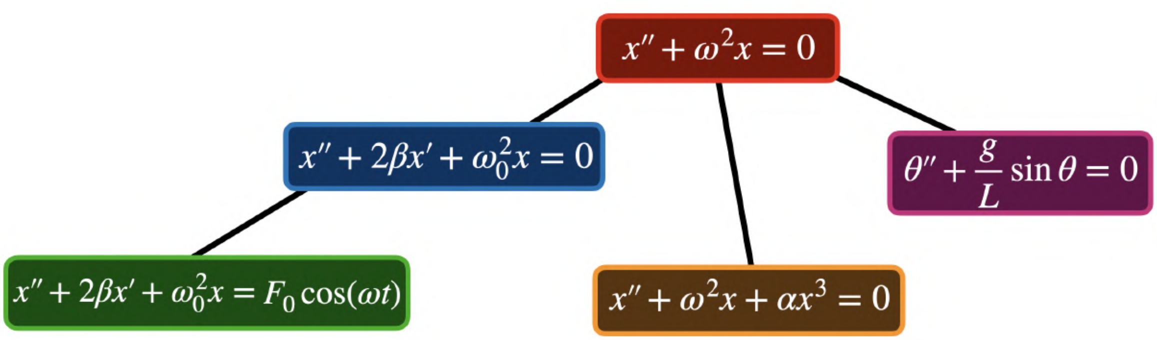
The Main **Oscillators** in Differential Equations



The Main Oscillators in Differential Equations

by DIBEOS





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The career of a young theoretical physicist consists of treating the harmonic oscillator in ever-increasing levels of abstraction.

Sidney Coleman

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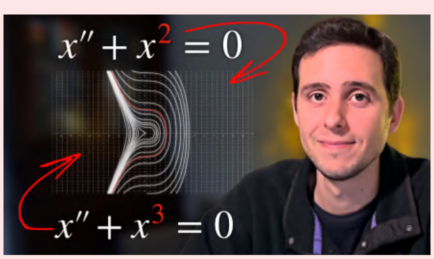
Suggestion



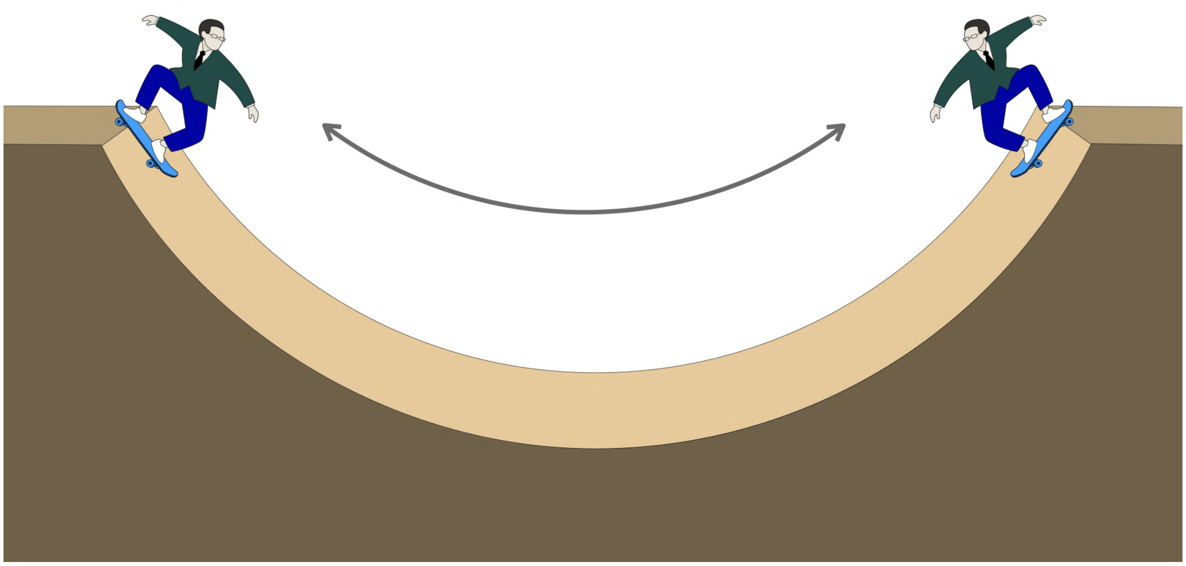
This material is a deeper look at the topics discussed in this YouTube video. We highly recommend watching the video first to get a basic understanding, and then reading this material. Click on the image.

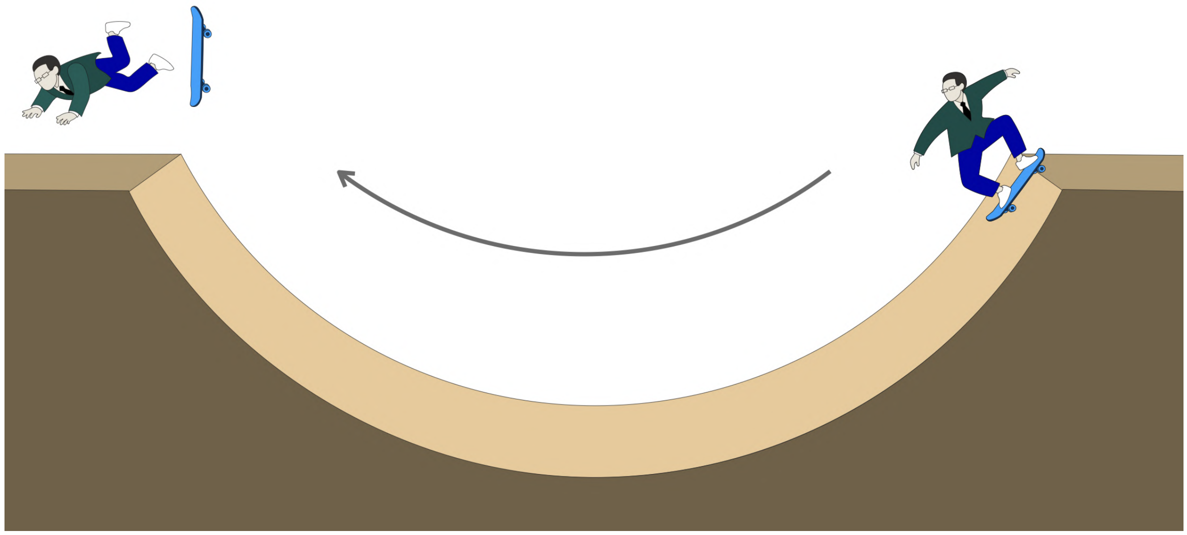
Introduction

Let me test your differential equations intuition for a second. Only one of the equations below describes a type of **oscillator**, the other one is an **impostor**. Can you tell which?

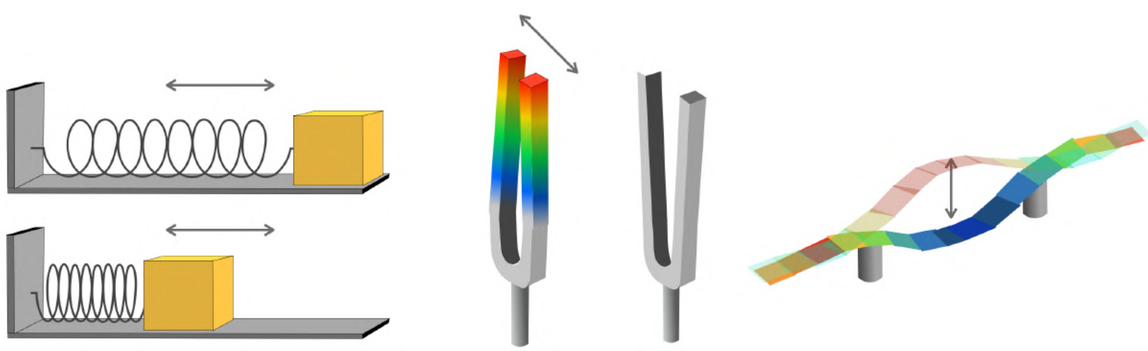
$$x'' + x^3 = 0 \qquad\qquad x'' + x^2 = 0$$

By the end of this material, you will understand why the correct one models periodic systems, while the wrong one might produce unstable situations instead.





If you guess correctly (and if there is some logic behind your reasoning, of course, if you are not just shooting blindly), then this is a sign that you have a good intuition on how differential equations work. But we need to start at the beginning.

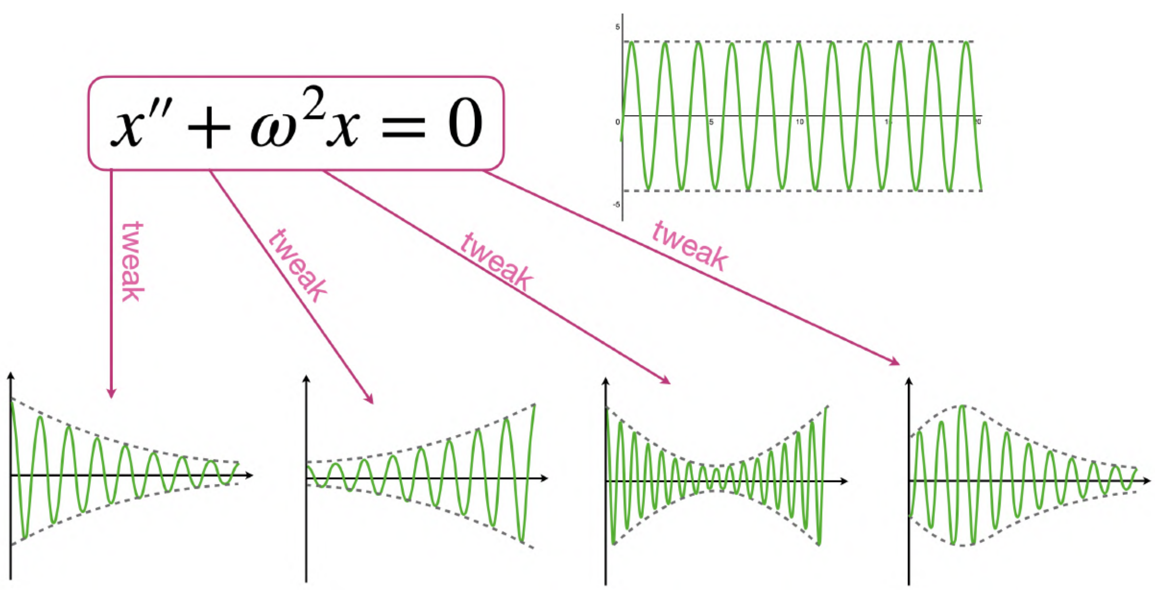


The **simple harmonic oscillator (SHO)** is the mathematical starting point for almost every oscillatory system out there. The solutions for this equation describe perfect sinusoidal motion.

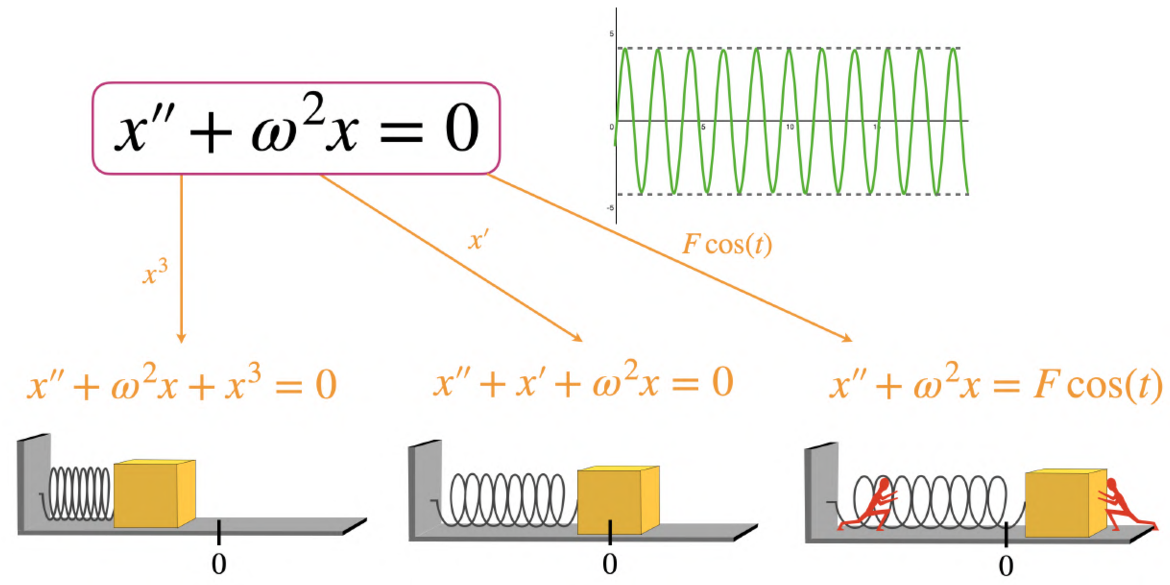
Historical Note

The concept of the simple harmonic oscillator emerged from early studies of periodic motion in the 17th century. **Galileo Galilei** observed that a pendulum's period is nearly independent of its amplitude for small oscillations. Later, **Christiaan Huygens** developed the mathematical description of pendulum motion in *Horologium Oscillatorium* (1673). These studies laid the foundation for the modern concept of the simple harmonic oscillator, which models systems where the restoring force is proportional to the displacement.

The problem is that in the real world systems are not really ideal. By tweaking this equation slightly you can generate an entire universe of different oscillators studied in pure and applied mathematics.



Some of these tweaks involve adding nonlinear terms like x^3, or introducing damping terms like x', or forcing terms like $F\cos(t)$, and so on.



In that sense, the simple harmonic oscillator is the *root* of most oscillatory differential equations, at least in applied mathematics.

When it comes to pure mathematics, let's say that mathematicians tend to be way more creative, but here we will focus only on the most important ones. And that is why the simple harmonic oscillator is first in our list.

CLICK HERE TO CONTINUE READING