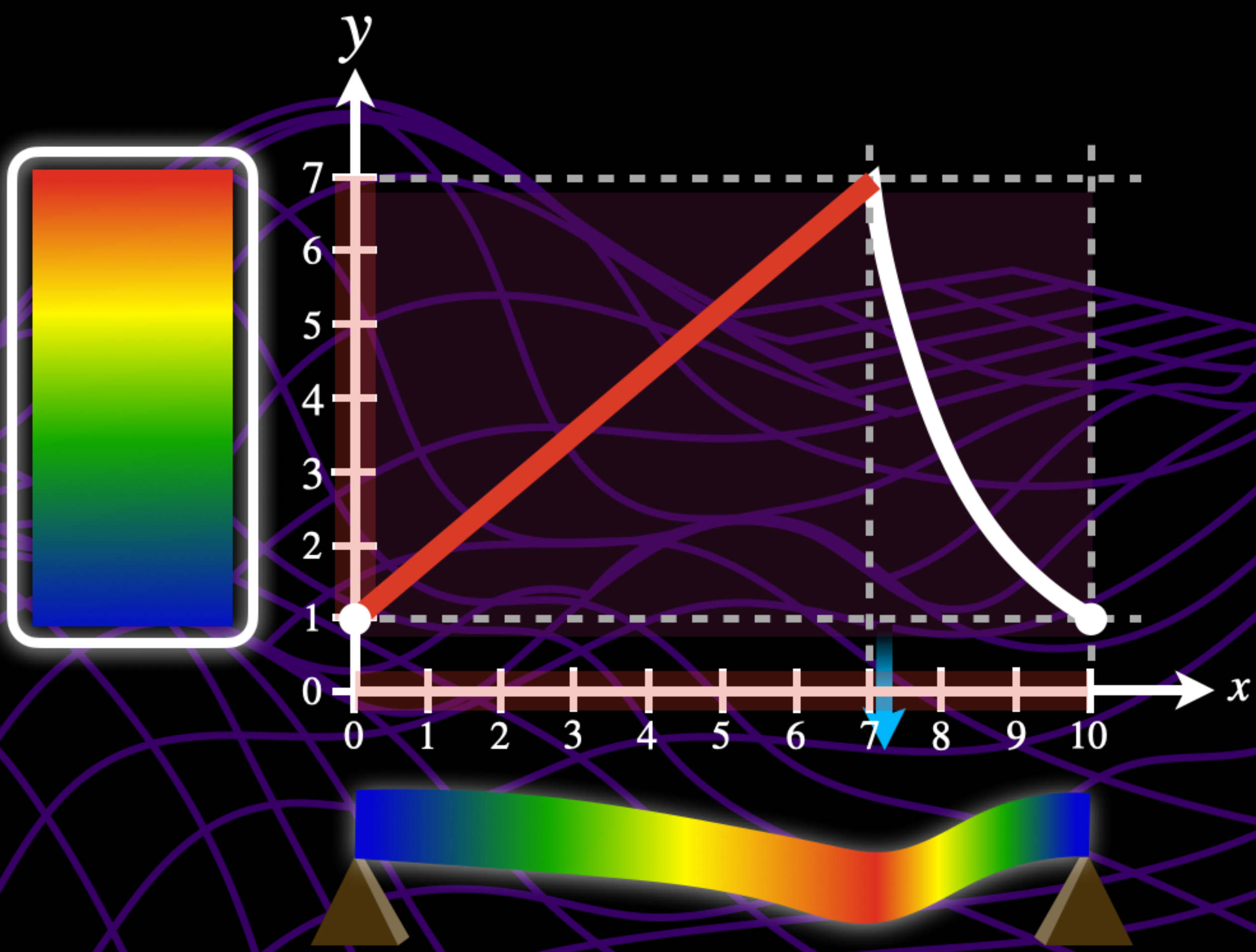
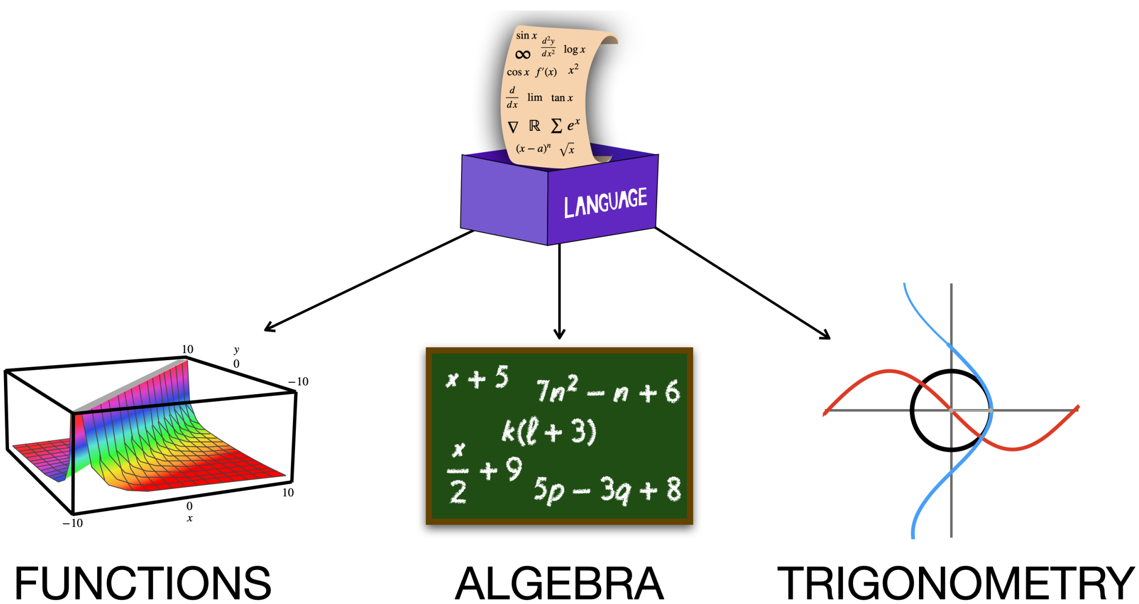
The Language of
Calculus



The Language of Calculus

by DIBEOS



Contents

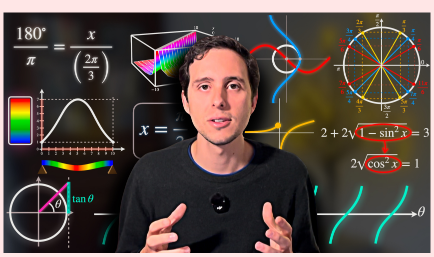
“

Calculus is the outcome of a dramatic intellectual struggle which has lasted for twenty-five hundred years.

Richard Courant

”

Suggestion



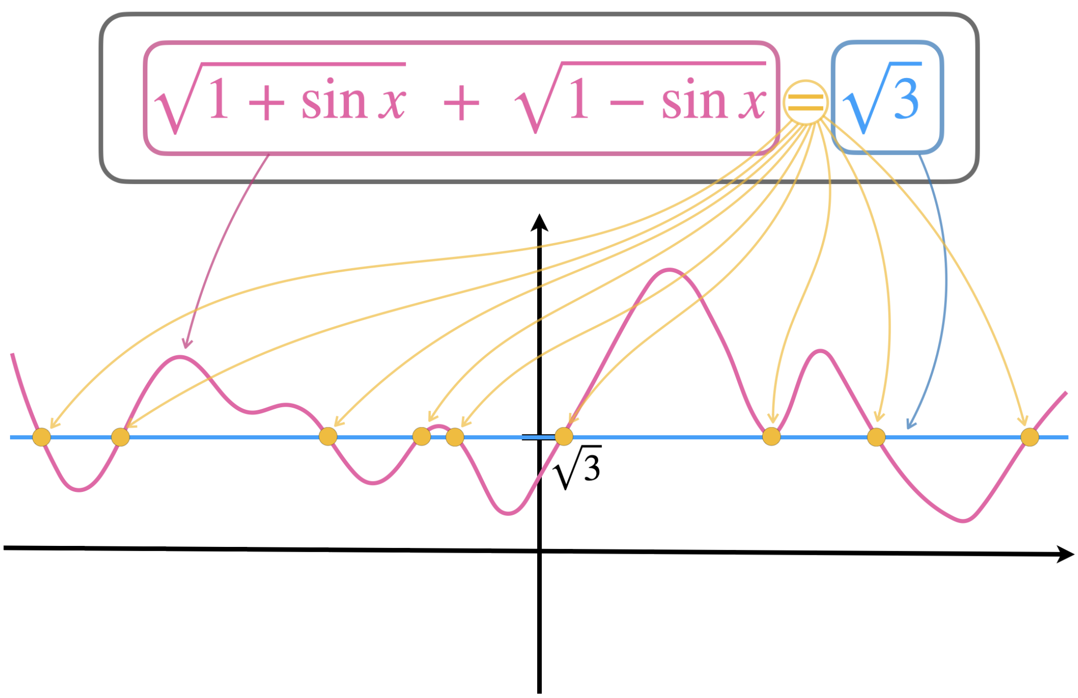
This material is a deeper look at the topics discussed in this YouTube video. We highly recommend watching the video first to get a basic understanding, and then reading this material. Click on the image.

Introduction

Take a look at the equation below and think to yourself: *"How would you go about solving it?"* You don't need to know Calculus to do it, but you do need a very strong foundation in the **Language of Calculus**.

$$\boxed{\sqrt{1+\sin x} + \sqrt{1-\sin x} = \sqrt{3}}$$

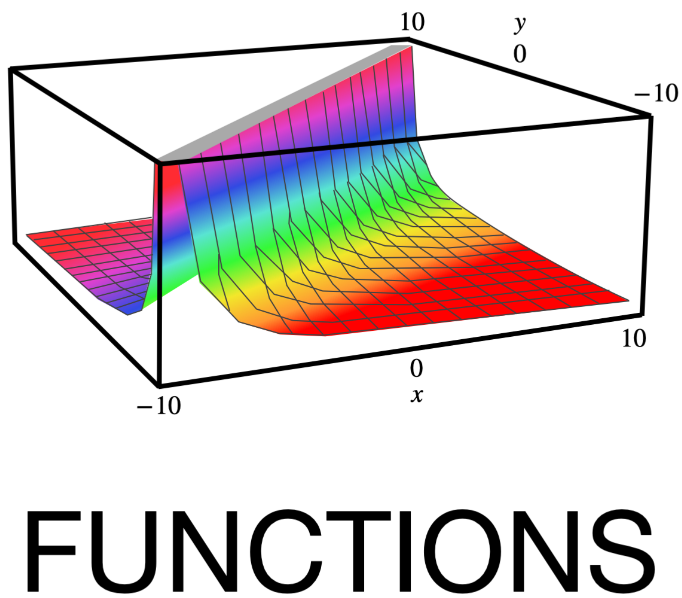
This is actually the main reason people struggle so much with Calculus I, II, III, and so on. They don't take the necessary time to build the right foundation.



That is what we are going to fix here. And after that, I promise you, mastering Calculus will be *easy peasy*.

FUNCTIONS

In order to solve this equation, you first need to master the concept of **functions**. Functions are the most important part of Calculus.

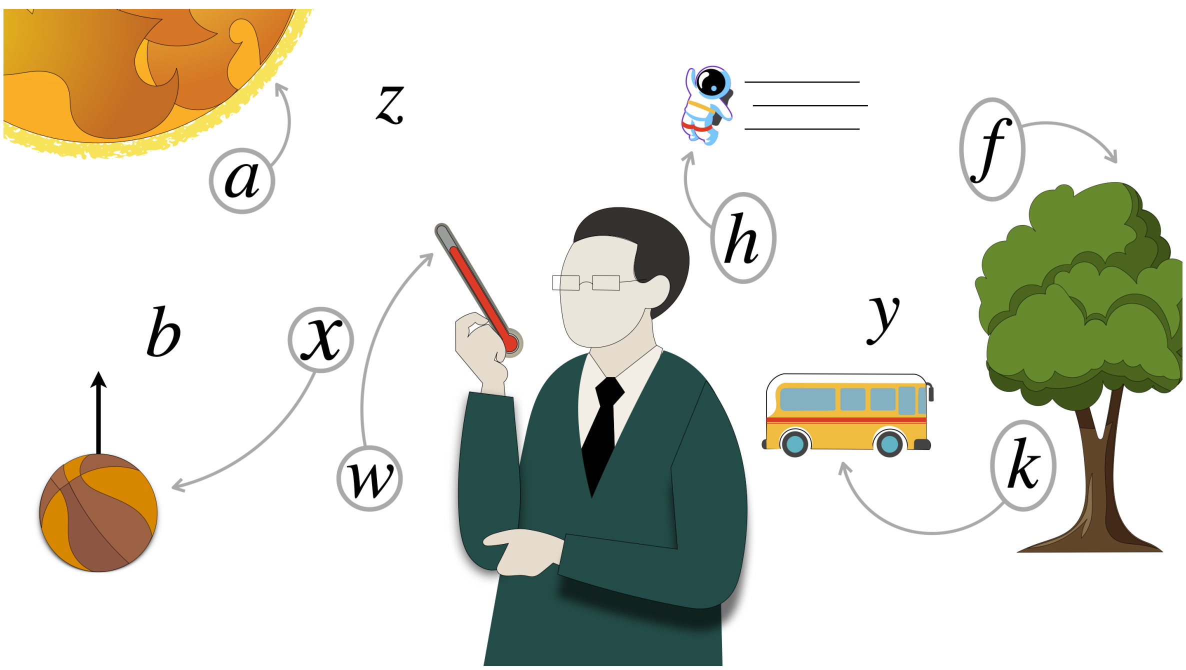


Out of all the skills you can build, deeply understanding functions, and having intuition on how they behave and how their graphs look is probably the highest-leverage thing you can focus on.

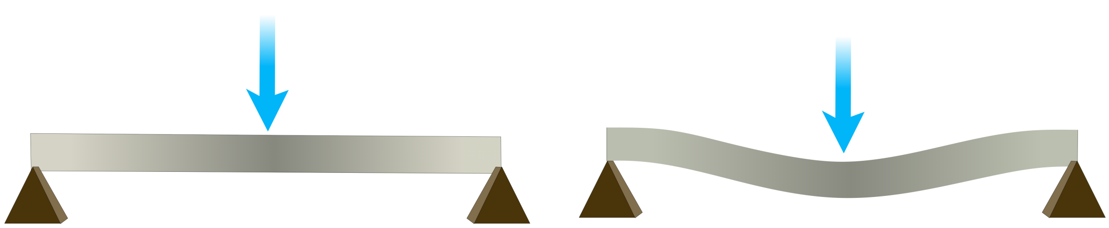
Nothing else will get you closer to mastering Calculus.

Variables & Dependency

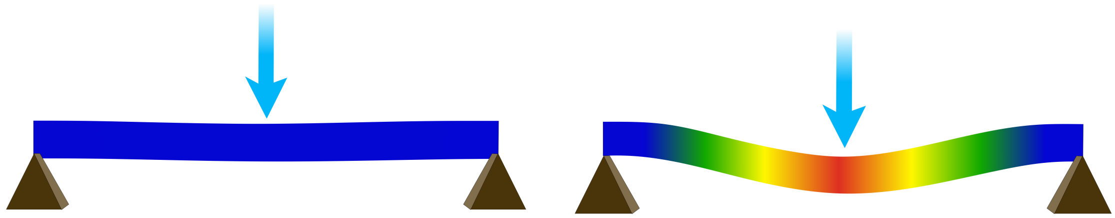
Let us start with a very simple definition of a function: a function is an object that relates two variables, and creates a dependency between them. Think about what a **variable** in your day-to-day life is. A variable is anything that you can measure and that varies.



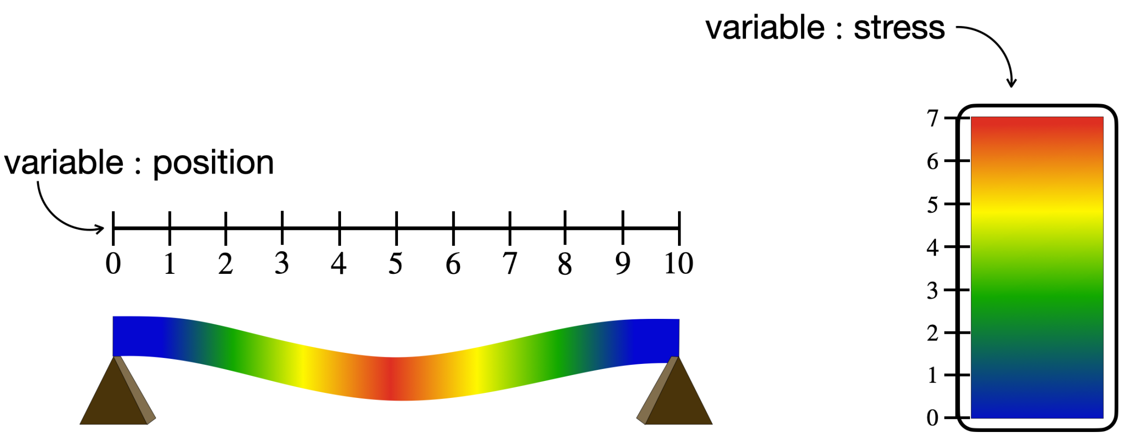
As an analogy, imagine a beam fixed at both ends. If you push it down, it bends. And as it bends, different points of the beam feel different amounts of stress.



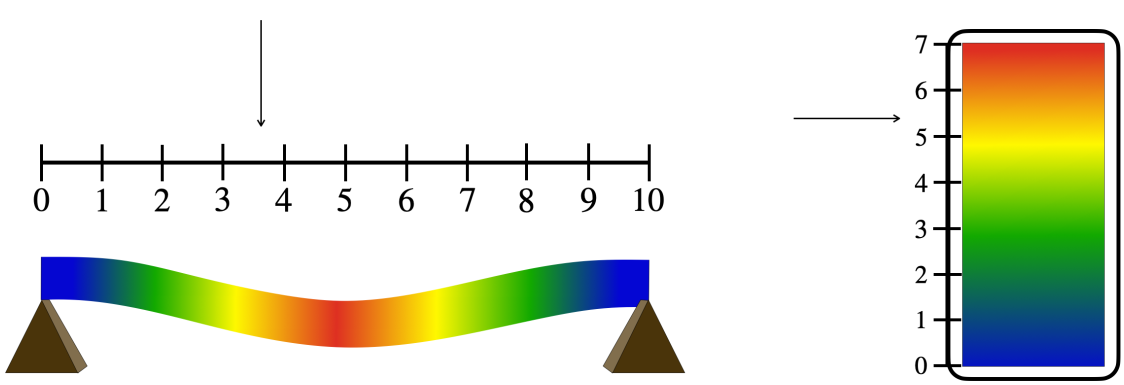
We will assign colors to those stresses such that each point of the beam gets its own color depending on how much it is being deformed.



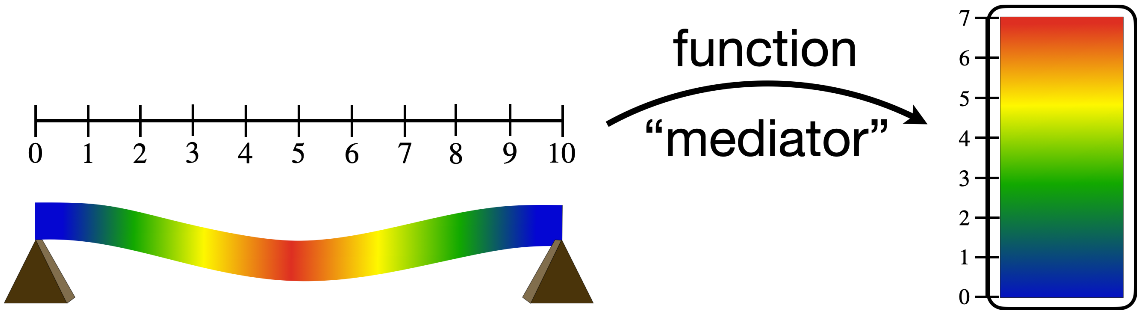
One of the variables is **position** along the beam, which we can measure with a ruler, and the other variable is **stress**, which we can measure with colors, or even better, by assigning a number to each color.



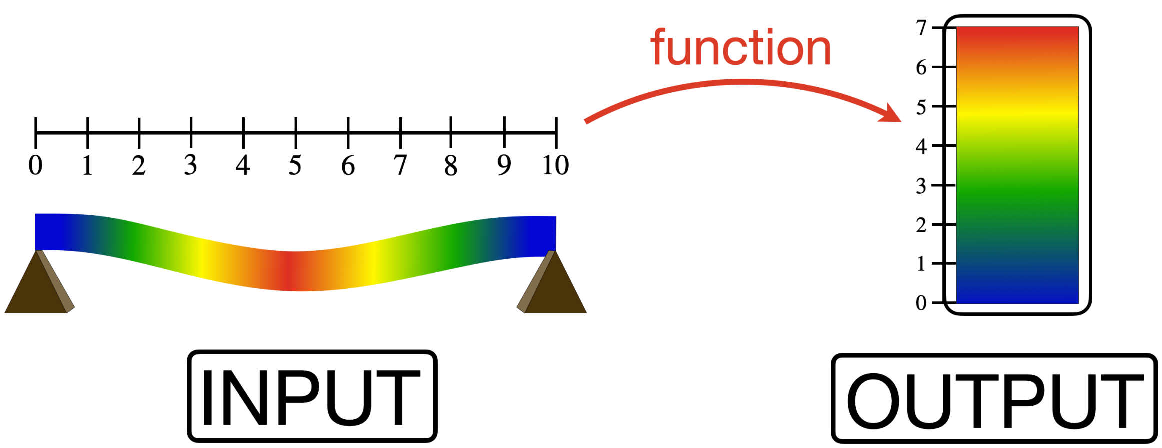
Notice how these quantities are continuous, so for example, we can measure positions 3.5 and 3.6, or even any position in between. We can measure stress corresponding to green (which is 4 in our scale), or yellow (which is 5), or even any level of stress in between.



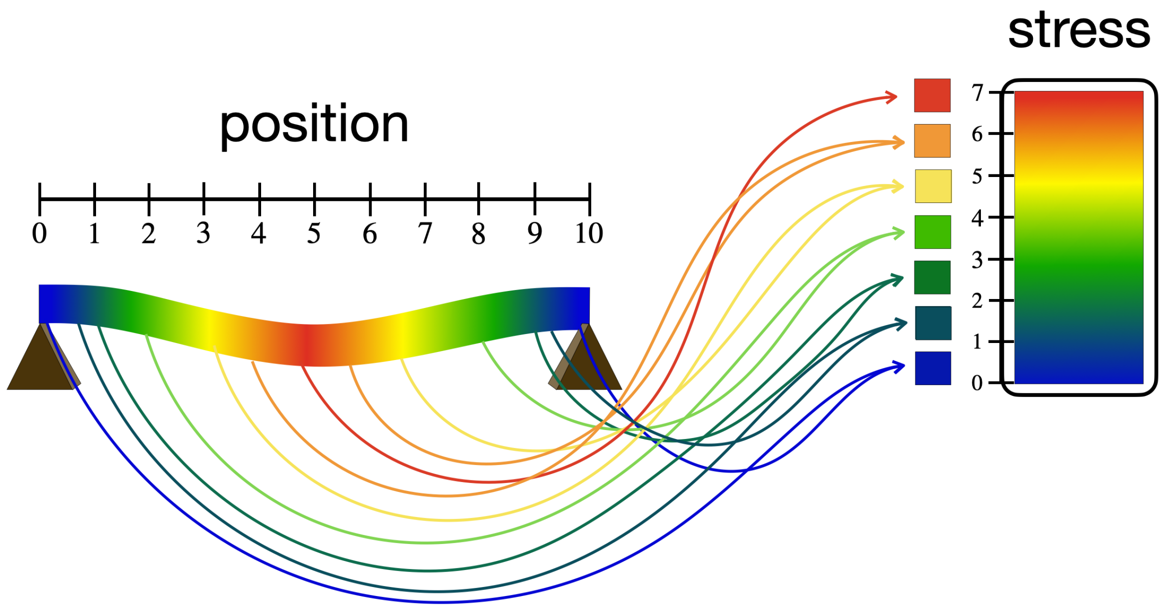
But, can you see how these measurements are not random? These variables are related to each other. The more we move from position 4 towards position 5, the more stress we measure in the beam. But if we continue moving right, we get less and less stress. So, these variables (position and stress) are not independent. There is some relation (some rule) between them. And (simplistically) that is the concept of a function. A function acts as a "broker", or a "mediator", between two variables.



Also, notice how there is a sense of "direction" for functions. In this case, we would say that the position is the **INPUT** value (i.e. the value given to the function), and its response, which is the level of stress, is the **OUTPUT** value.



For each position there is a stress assigned to it, but the opposite is not always true. The dependence goes in a specific direction: each position must give exactly one stress value.



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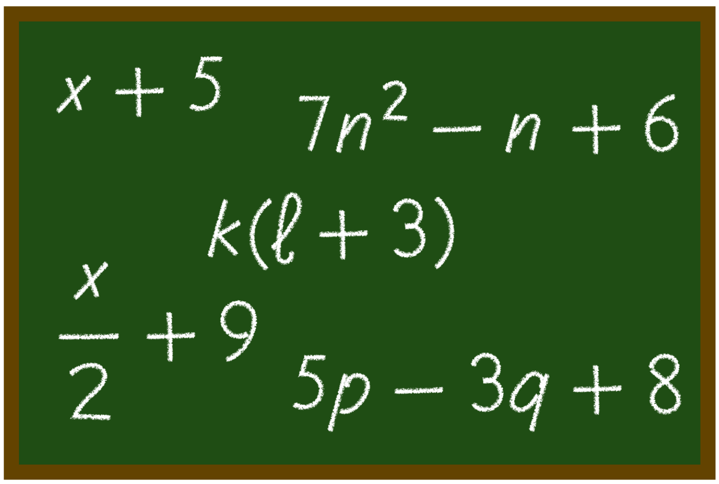
ALGEBRA



ALGEBRA

Specifically for Calculus, what you need from algebra is the ability to manipulate expressions and solve **equations**, **systems of equations**, **inequalities**, and **systems of inequalities**.

equations : $x^2 + 3 = -x$

systems of equations : $\begin{cases} x^2 + 8y = -2 \\ y = -x^3 \end{cases}$

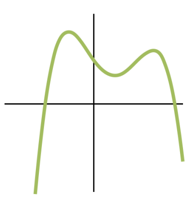
inequalities : $-x^2 + 3x + 2 \leq 0$

systems of inequalities : $\begin{cases} x + 3 < 0 \\ -x^2 + 1 \geq 0 \end{cases}$

BUT! The problem is that inside of each of these branches, we can have all the functions we saw before (see below), plus some creative combinations of them.

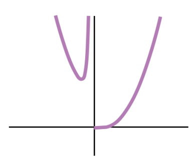
Polynomials

$y = ax^4 + bx^3 + cx^2 + dx + e$



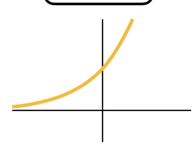
Rational Functions

$f(x) = x^2 \cdot e^{-\frac{1}{x}}$



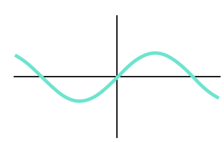
Functions Exponential

$y = 3^x$



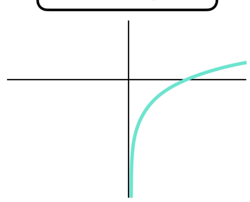
Trigonometric Functions

$y = \sin x$



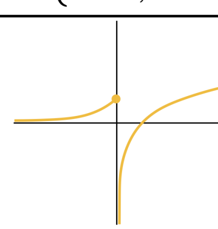
Logarithmic Functions

$y = \log(x)$



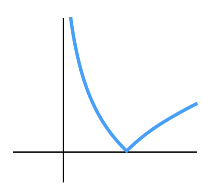
Piecewise Functions

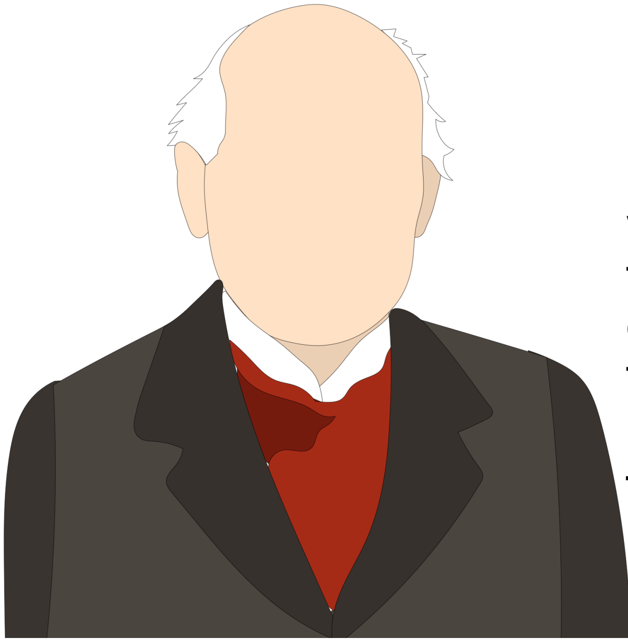
$y = \begin{cases} e^x & , \quad x \leq 0 \\ \ln x & , \quad x > 0 \end{cases}$



Absolute Value Functions

$y = |\ln(x)|$





"Algebra is the intellectual instrument which has been created for rendering **clear** the quantitative aspects of the world."

– Alfred North Whitehead

The Goal of Algebra

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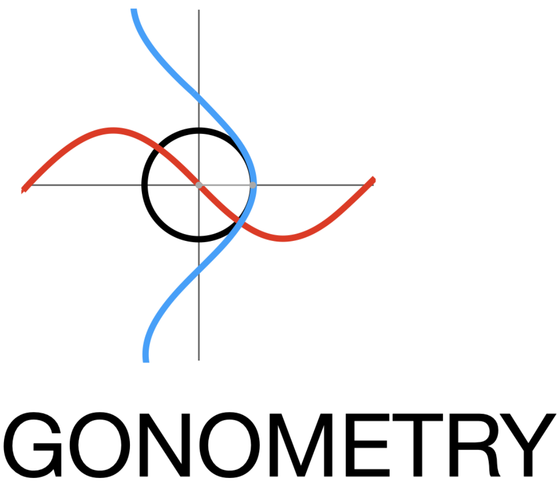
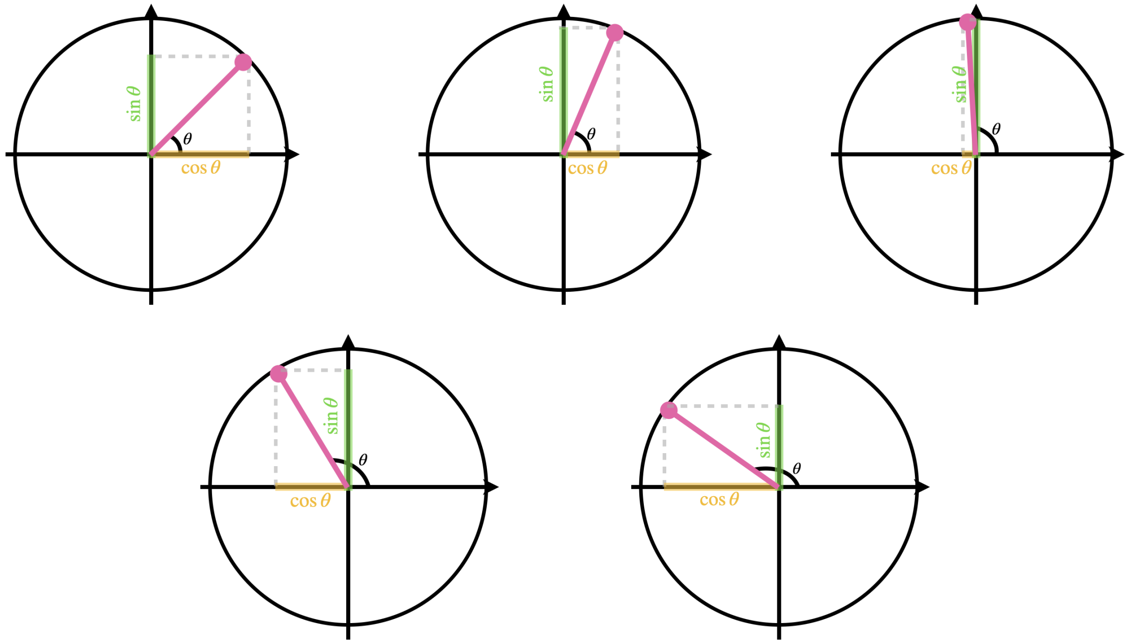
TRIGONOMETRY



"**Trigonometry** is **not about triangles**, it is about **rotations**!"

cyclic & periodic behavior

The unit circle is the main object of study here, it just turns out that triangles naturally appear when we try to describe the position of a rotating point along the unit circle using horizontal and vertical coordinates.



Rotations & The Unit Circle

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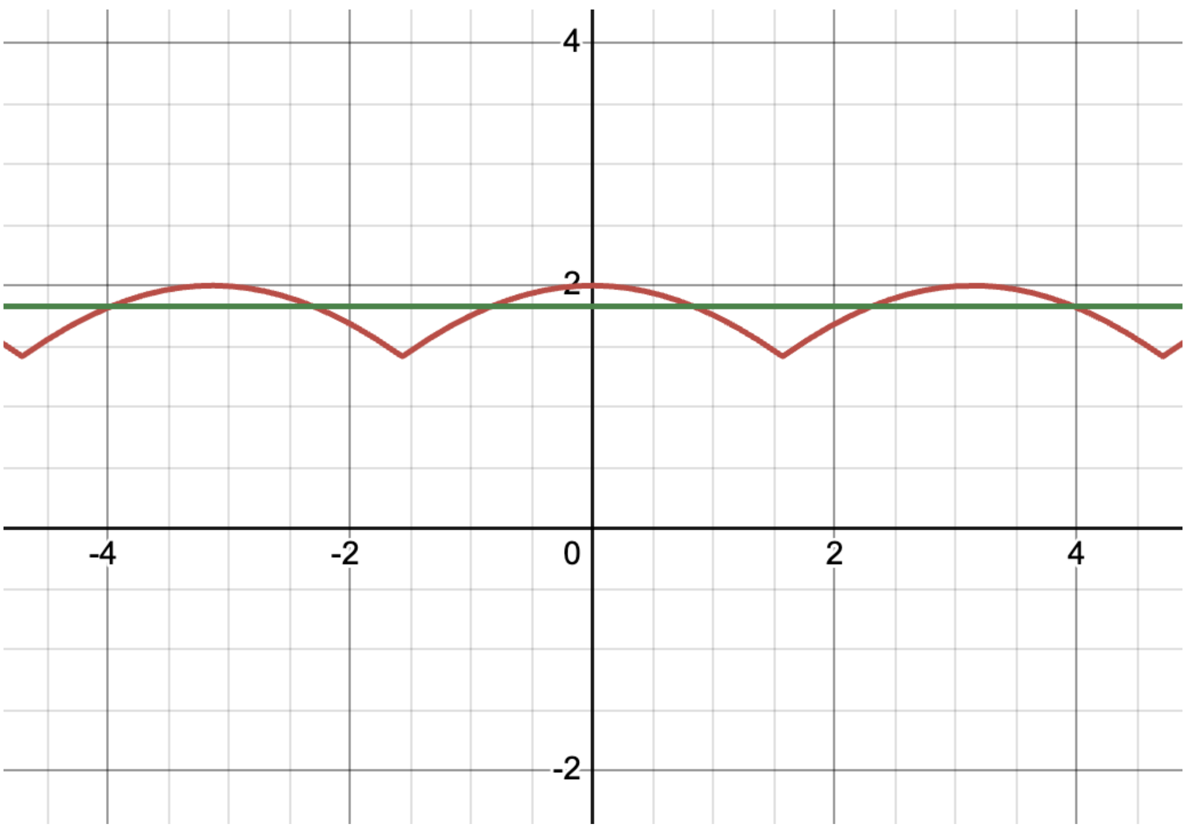
Bonus: INFINITESIMALS

$$\boxed{\sqrt{1+\sin x}+\sqrt{1-\sin x}=\sqrt{b}}$$

We can slightly change the right-hand side of the equation by introducing a parameter $b \in \mathbb{R}_+$.



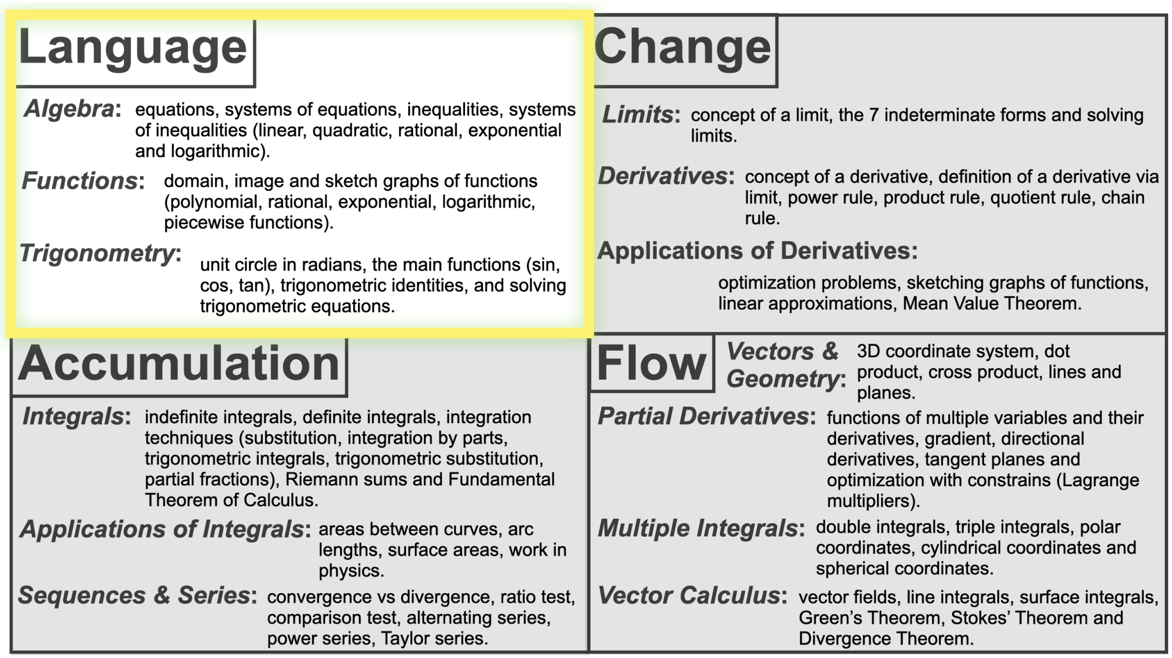
Now, my challenge for you is to figure out for which values of b this equation has solutions. Think about it for a moment before seeing the solution below.

Solution

Conclusion

Summary

You just finished this section:



You only have 3 more to go. Come on!

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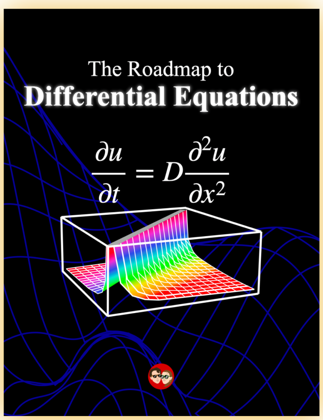
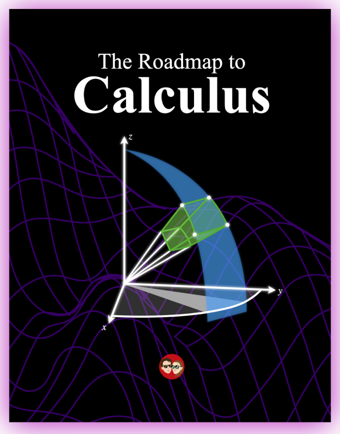
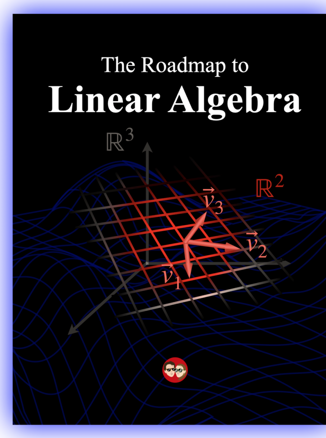
Do not forget to check out our catalogue of learning material. You might find something interesting. Just click on one of these covers:







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