

Solving 1st-Order Ordinary Differential Equations



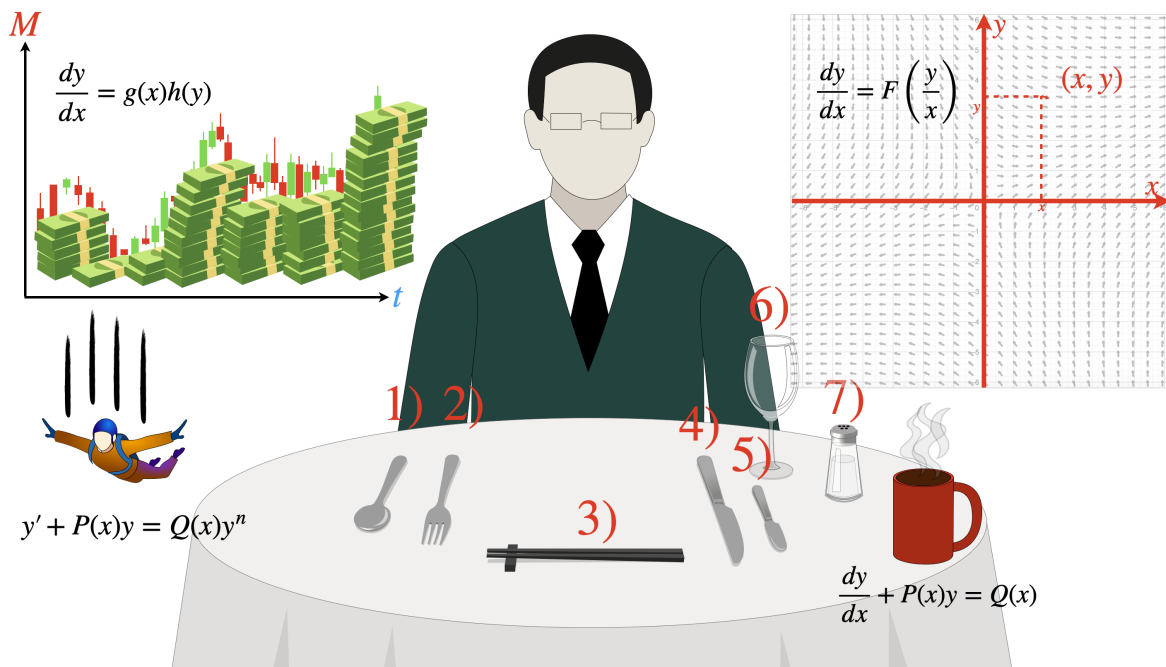
$$y' = a(x)y^2 + b(x)y + c(x)$$





Solving First-Order Ordinary Differential Equations

by DIBEOS



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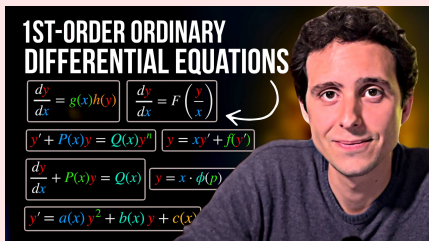
“

In order to solve a differential equation,
you look at it till a solution occurs to you.

George Pólya

”

Suggestion

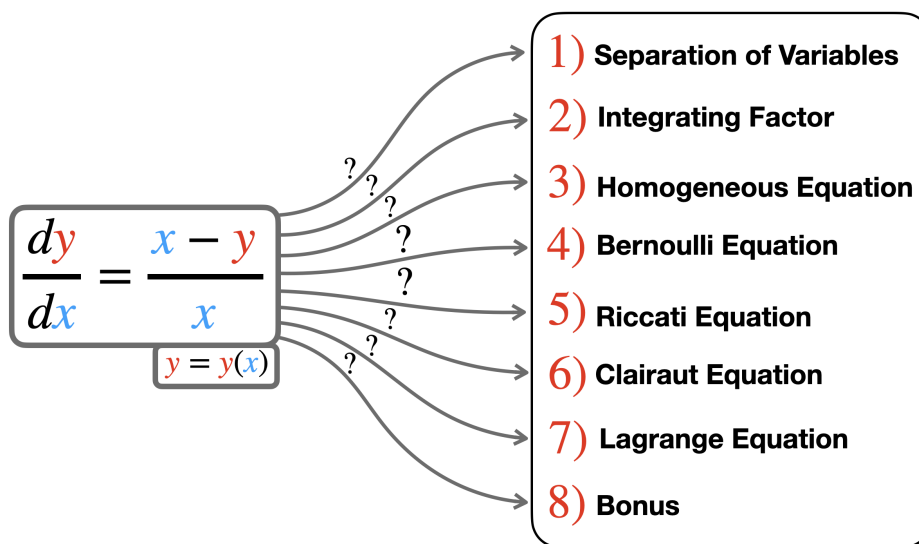


This material is a deeper look at the topics discussed in this YouTube video. We highly recommend watching the video first to get a basic understanding, and then reading this material. Click on the image.

Introduction

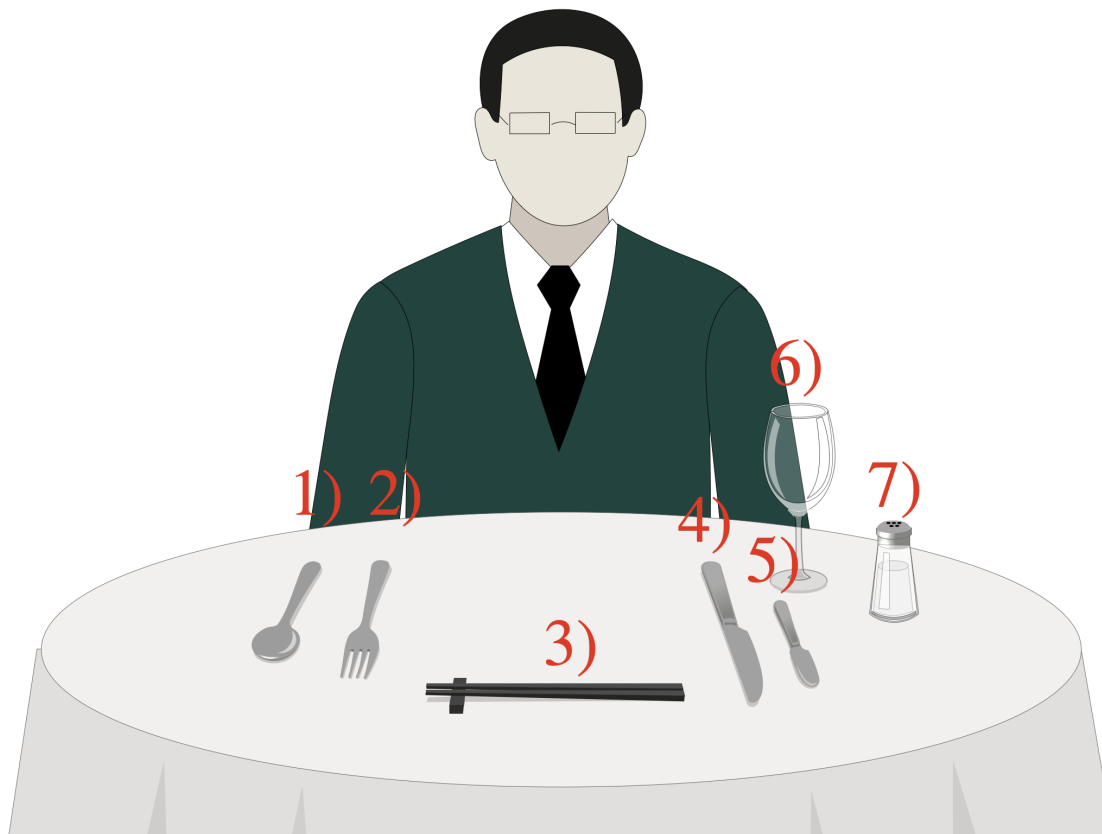
$$\frac{dy}{dx} = \frac{x - y}{x}$$
$$y = y(x)$$

Look at the differential equation above. My *challenge* for you is to decide which one is the best method to solve it, based on its structure. It has a very common structure, but only a person with trained eyes can recognize the pattern and can tell what the method is.



In this material, we will see the 7 most important techniques you can use to solve **first-order ordinary differential equations**, like this one.

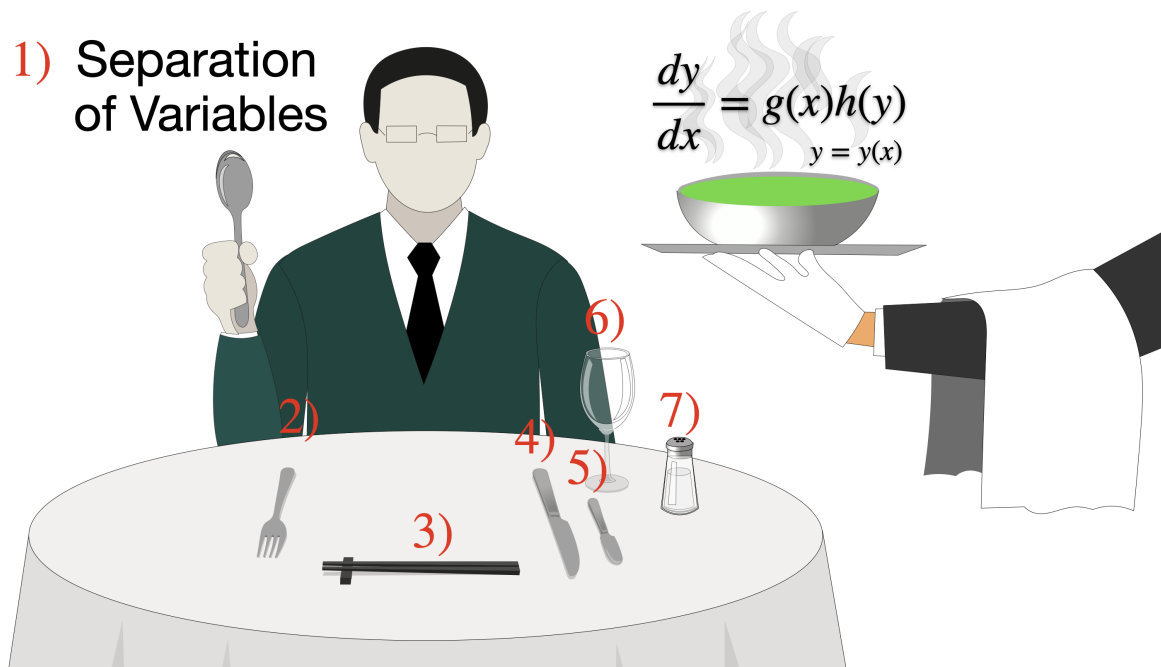
Just as when we go to a restaurant we have to pick different tools to eat different kinds of dishes and drink different kinds of beverages, when a differential equation is handed to us, we need to match it with the right method, so that we can solve it.



Let us now see the 7 different structures. Your goal is to use your *pattern recognition* skills to spot which one best fits this equation.

Separation of Variables

1) Separation of Variables



The first method is **Separation of Variables**. The following is the general structure that you have to spot in order to use this method:

$$\frac{dy}{dx} = g(x)h(y)$$

Separation of variables is the most natural method to use in order to solve this type of differential equation.

y – side x – side

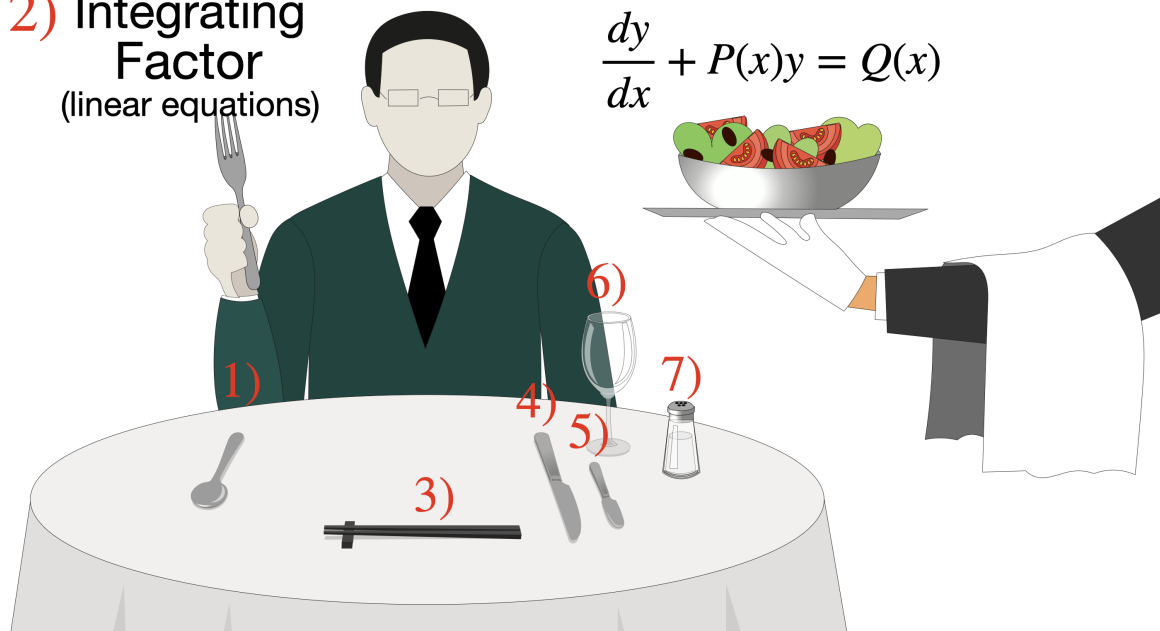
$$\frac{dy}{h(y)} = g(x) dx$$

Example: Financial Investment

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Integrating Factor

2) Integrating Factor (linear equations)



The second method is: **Integrating Factor**. It is used when we have a *linear differential equation*. The idea here is to take advantage of the product rule of derivatives.

$$\frac{dy}{dx} + P(x)y = Q(x)$$

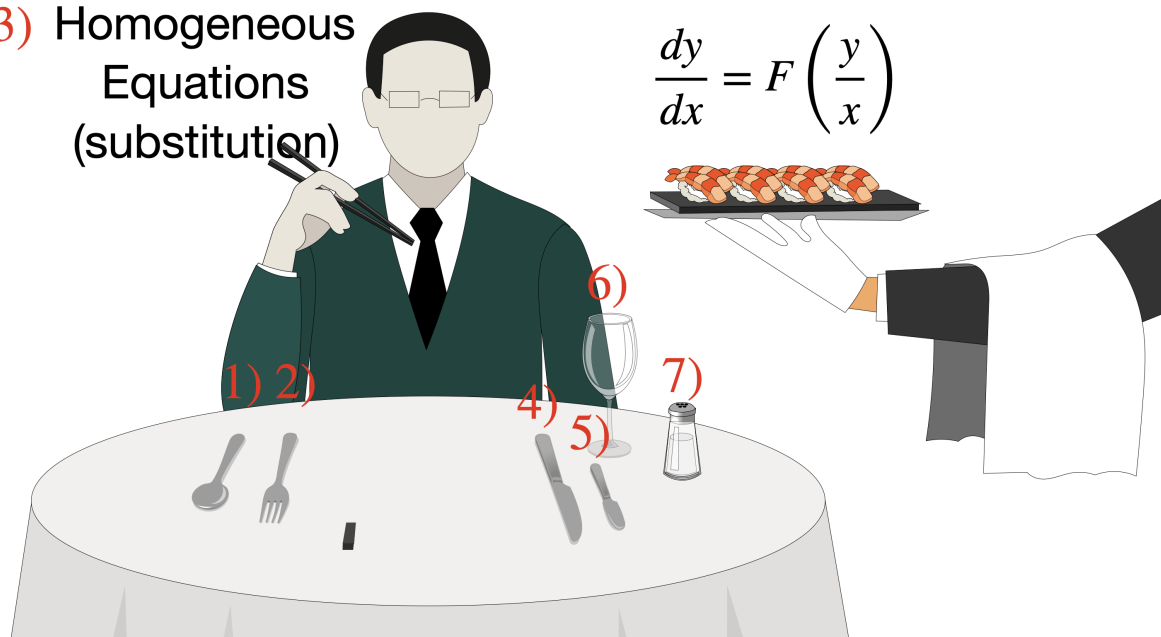
product rule of derivatives: $(f \cdot g)' = f' \cdot g + f \cdot g'$

We basically want to force the left-hand side of the equation into a *product derivative*.

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Homogeneous Equations

3) Homogeneous Equations (substitution)

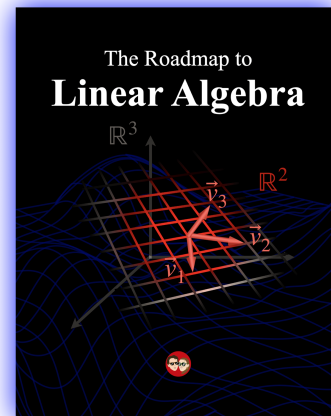
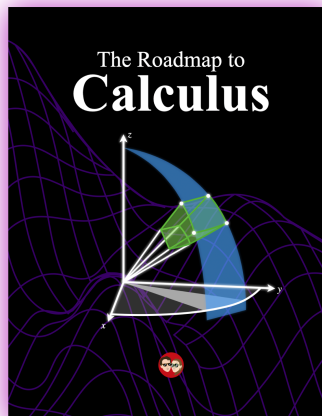
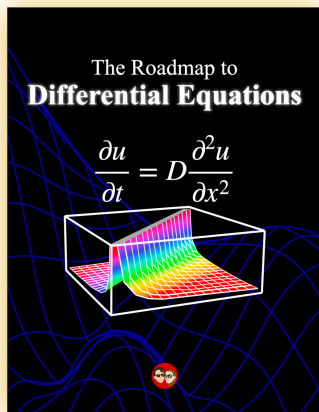


Homogeneous means “*same structure under scaling*”¹. This method consists of a clever *substitution* that simplifies homogeneous differential equations. But first, let us see examples of homogeneous functions:

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¹ $f(\lambda x, \lambda y) = \lambda^k f(x, y)$.

Do not forget to check out our [catalogue](#) of learning material. You might find something interesting. Just click on one of these covers:



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